Technological transfers, limited commitment and growth

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Abstract

This paper examines the effect on economic growth and welfare of the access to external financing which results in technological transfers to a developing country from the rest of the world. We study a two-sector stochastic growth model and compute optimal accumulation mechanisms in the environments which differ in the extent to which the borrowing contracts are enforced. Furthermore, we examine different assumptions concerning the default punishment and their implications for growth, welfare and borrowing patterns. We show that under limited commitment lack of technological transfers may result in scarce capital flows to developing countries and substantially reduce their growth opportunities. Presence of the technological transfers in this environment induces a developing country to use foreign capital to both smooth consumption and invest more heavily in all the sectors of the economy including those directly unaffected by the productivity benefits. Our findings suggest that technological transfers may play a role of an important enforcement mechanism. In addition, our model can account for the rich structure of observed capital flows to low- and middle income countries.

*Journal of Economic Literature* Classification: C63, F34, O33, O40.

*Keywords:* Incentive compatibility, technological diffusion, international capital flows, default risk, numerical algorithm.

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1 Introduction

The purpose of this paper is to study the effect on economic growth and welfare of the access to external financing which results in technological transfers to a developing country from the rest of the world. We study a stochastic growth model with two productive sectors one of which may be affected by the productivity spillovers from abroad. We focus on the institutional aspects of the economy and compute optimal accumulation mechanisms in the environments which differ in the extent to which the borrowing contracts are enforced. We examine the role of two alternative assumptions about the severity of the default punishment and their implications for growth, welfare and borrowing patterns. Our results show that under limited commitment lack of technological transfers may result in scarce capital flows to developing countries and substantially reduce their growth opportunities. On the other hand, presence of technological transfers in this environment induces a developing country to use foreign capital to both smooth consumption and invest more heavily in all the sectors of the economy including those directly unaffected by the technological diffusion. Our findings suggest that technological transfers may play a role of an important enforcement mechanism. In addition, our model outperforms the standard models of economic growth in its ability to account for the rich structure of observed capital flows to low- and middle income countries.

Conventional growth models have been reported to face certain difficulties in accounting for the observed pattern of capital flows from the industrialized to the low- and middle income countries. These difficulties have often been shown to stem from the failure to model certain margins. For instance, Barro et al (1995) discussing international capital mobility emphasize that some forms of capital cannot be financed by borrowing on the world market. They show that an open-economy version of the neoclassical model is no longer at odds with empirical evidence on convergence once capital is viewed broadly to include human investments. The role of institutional or contractual elements is stressed by, among others, Marcet and Marimon (1992) and Thomas and Worrall (1994). Marcet and Marimon (1992) show that enforcement constrains may result in negligible transfers to a developing country and severely reduce its growth opportunities. The objective of this paper is to follow the latter line of research by studying optimal accumulation mechanisms in the environments with different degree of enforcement of contracts.

One novelty of our framework is that the access to the external financing is assumed to result in technological transfers to the developing countries from the rest of the world. This assumption finds extensive empirical support in the literature. Some of the studies emphasize the positive effect on productivity of free capital movement as such. For example, Frankel and Romer (1999) conclude that the benefits from integration for a developing country partially
stem from the transfer of ideas from the rest of the world. In line with that *Global Development Finance* (World Bank, 2001:59) states that there is ample evidence indicating towards the productivity benefits of the capital flows "through transfer of technology and management techniques".

Other studies stress the importance of foreign direct investment (FDI) as a mechanism of technological transfers to the developing countries from the rest of the world. For instance, according to World Bank (2001), depending on the absorptive capacity, FDI has been positively associated with the productivity of the foreign owned firms and with positive spillover to domestically owned firms. Romer (1993) suggests that FDI has considerable potential to transfer ideas from the industrialized countries to the developing countries. Görg and Strobl (2001) provide a comprehensive review of the empirical literature on FDI and productivity spillovers. They also give account of other channels through which productivity spillovers may occur such as movement of highly skilled personnel, the 'demonstration effect' or the 'competition effect'. FDI as a potential mechanism of technological transfers has been particularly emphasized due to its increasing role in the stream of international capital flows to low- and middle income countries. As documented by Thomas and Worrall (1994) already in the mid-eighties about a half of all capital flows to the developing countries took form of FDI. The fraction of FDI in the international capital flows kept increasing during the last two decades and now it constitutes the most important net flow for all regions (Blitzer, 2003).

The setup of our framework is similar to that of the full information environment of Marcet and Marimon (1992). We study a model with two agents, one risk-averse agent representing a developing country and the other risk neutral agent representing the rest of the world. There is no disutility from labor. We focus on the growth of the developing country which is assumed to have low initial level of capital. In this context, growth is understood as a transition from the initial low level of capital to the steady state. We analyze the model within three environments which differ to the extent the debt contracts are being enforced. These are: (i) autarky; (ii) external financing with perfect enforcement of contracts; and (iii) external financing with limited enforcement of contracts. Under the latter regime, a developing country may at any moment appropriate the accumulated capital and refuse to honor its debt. In this case it will suffer a default punishment which will involve loss of any external financing opportunities in the future.

We assume that there are two productive sectors in the economy, which we refer to as domestic and foreign operated sector. Each of the sectors has Cobb-Douglas technology. The risk averse agent decides how much to invest in each of the sectors. The technology which converts investment into capital goods is non-linear and affected by the productivity shocks. The foreign sector is assumed to be more productive due to technological transfers associated
with external financing. This assumption relies on the literature review by Görg and Strobl (2001) who document that in the literature it is often argued that the positive spillovers only affect certain sectors of the economy. As in Cohen and Sachs (1986) or Eaton and Gersovitz (1984) failure to honor the foreign debt results in permanent loss of productive efficiency.

We consider two modifications of the model which differ in the default punishment a developing country will endure should it refuse to honor its debt contracts. First, we analyze a model where in case of debt repudiation the country loses not only productivity benefits in the foreign operated sector but also accumulated capital in this sector. Furthermore, the country is deprived from the possibility to develop this sector on its own. Such assumption is used for instance by Marcet and Marimon (1998) where they study a partnership with limited commitment. Under this assumption, the autarkic environment, which is hereafter referred to as one-sector autarky, is similar to the stochastic growth model of Brock and Mirman (1972) augmented with non-linear stochastic investment technology. Our key finding from this model is that technological gains from external financing opportunities may eliminate the default risk even though they affect only some sectors of the economy.

The discussed above assumption of the punishment is case of deviation from the optimal plan may be judged as extremely severe. Indeed, the defaulting country loses not only all the productivity benefits and capital accumulated in the foreign operated sector but also a possibility to develop this sector on its own. Therefore, we consider a framework where in case of debt repudiation the developing country looses the technological advantage associated with external financing. However, the country appropriates capital stock in all sectors of the economy which still remain productive with the productivity level of the domestically operated sector. Relying on this assumption we consider three representative cases which differ in the extent of the technological diffusion. These are: the no transfers case; the medium transfers case, which is characterized by the presence of the default risk; and the high transfers case where the risk of default is eliminated.

Our findings allow to conclude that the existence of substantial capital flows from the developed to developing countries is not inconsistent with the presence of the default risk. We overcome the difficulty that the models of sustained growth have in explaining the rich structure of observed capital flows and borrowing patterns across low- and middle-income countries. Our framework suggests that under limited enforcement the pattern of capital movements depends heavily on the productivity benefits associated with the external financing opportunities.

We also conclude that technological transfers may play a role of an enforcement mechanism. In our framework even moderate technological benefits associated with external financing opportunities may substantially reduce the negative effect on the welfare of the failure
to perfectly enforce contracts. Presence of technological diffusion in the environment with limited commitment induces a developing country to use foreign capital to both smooth consumption and invest more heavily in all the sectors of the economy including those directly unaffected by the technological transfers. The latter results in faster growth and significant welfare gains.

Since we study models with participation constraints, which involve expected values of the future variables, we are unable to use the results of standard dynamic programming. Our methodology relies on the contribution of Marcet and Marimon (1998) who have demonstrated that problems with incentive compatibility constraints fall into a general class of problems, which can be cast into alternative recursive framework. Our numerical analysis utilizes the parameterized expectation approach (PEA) introduced by Marcet (1989). Although PEA algorithm approximates the true equilibrium at the steady state distribution with arbitrary accuracy, the policy function obtained from the long-run simulations may not be a good approximation for the solution during the initial periods. This is of particular importance for our analysis since we study growth of the economy during the transition towards the steady state. To overcome this problem we adapt a numerical algorithm introduced by Marshall (1988) and developed by Marcet and Marimon (1992) to find a distinct policy function for the initial periods.

Our paper is most closely related with the studies of Marcet and Marimon (1992) and Maliar et al (2004). Indeed, in many aspects our setup mimics that of the full information environment of Marcet and Marimon (1992). In this sense, our framework may be considered as an extension to their model which allows for the technological transfers affecting some sectors of the economy.

The rest of the paper is organized as follows. Section 2 presents the baseline models corresponding to the three environments: one-sector autarky, external financing with full and limited enforcement. These models rely on the most stringent assumption about the default punishment. Section 3 describes the numerical algorithms for solving the models and analyzes the solutions for them. Section 4 introduces the model with two-sector autarky which relies on a more moderate assumption concerning the default punishment. Section 5 analyzes the numerical solutions corresponding to the models with two-sector autarky which differ in the magnitude of technological transfers. Section 6 concludes.

2 The Baseline Model

The environments considered in the paper essentially share some features. There are two agents: agent 1 who is risk averse and can be interpreted as a developing country and agent
2 who is risk neutral and represents the industrialized countries. As in Marcet and Marimon (1992) the technologies that convert investment into capital are non-linear and are affected by a productivity shock.

2.1 Efficient growth mechanism under full commitment

It is assumed that there are two sectors in the economy which will be called domestic and foreign operated sector. In the case of external financing due to technological transfers the foreign operated sector will enjoy higher productivity as compared with the domestic sector. This assumption can be justified along the following lines. The technological transfers partially originate from the fact that a part of capital inflows into a country will take form of FDI. It is often argued in the literature that the positive spillovers from FDI only affect certain firms in the domestic economy (Görg and Strobl, 2001). The set of firms which are affected by the technological transfers from the rest of the world will be referred to as foreign operated sector.

In this environment, the efficient growth mechanism, \( \Gamma \), represents a state-contingent investment and transfer plans \( \Gamma = \{i_{1t}, i_{2t}, \tau_t\} \) which is obtained as a solution to a dynamic principal-agent problem for a given set of initial conditions and weights. The latter are comprised of the initial capital stocks \( k_{10}, k_{20} \), the initial productivity shock \( \theta_0 \), and the weight \( \lambda \in \mathbb{R}_+ \) assigned to the risk-averse agent in the planner’s problem given by

\[
\text{Program 1} \\
\max_{\{c_{1t}, \tau_t, i_{1t}, i_{2t}\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \lambda u(c_{1t}) + (-\tau_t) \right] \right] \\
\text{subject to} \\
c_{1t} - \tau_t + i_{1t} + i_{2t} = f(k_{1t}) + F(k_{2t}), \\
k_{1t+1} = (1 - \delta)k_{1t} + g(i_{1t}, \theta_{t+1}), \\
k_{2t+1} = (1 - \delta)k_{2t} + g(i_{2t}, \theta_{t+1}),
\]

with \( c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0 \) given.

In this specification \( u(\cdot) \) represents the instantaneous utility of the risk-averse agent. We denote as \( f(\cdot) \) and \( F(\cdot) \) the production functions corresponding to the domestic and foreign operated sectors of the economy. The function that transforms units of investment into units of capital is denoted as \( g(\cdot) \). The consumption of the risk-averse agent is given by \( c_{1t} \), the transfers from the risk-neutral agent to the risk-averse one are denoted by \( \tau_t \). Investment in to the two sectors are given by \( i_{1t} \) and \( i_{2t} \), and the corresponding capital stocks by \( k_{1t} \) and
The variable $t_{t+1}$ represents an exogenous stochastic shock, the realization of which is unknown at the time the investment decisions are made.

The following assumptions, relatively standard in the stochastic growth literature, will hold throughout the rest of the paper:\(^1\): (i) the utility function $u(\cdot)$ of the agent 1 is strictly concave, twice differentiable and satisfies the Inada conditions: $\lim_{c \to 0} u'(c) = +\infty$, $\lim_{c \to \infty} u'(c) = 0$; (ii) the sectorial production functions $f(\cdot)$ and $F(\cdot)$ are concave and differentiable; (iii) the exogenous stochastic process $\theta_t$ is stationary and has bounded support; (iv) depreciation rate $\delta \in [0, 1]$; (v) $g(\cdot, \theta)$ is differentiable and concave.

A note on the interpretation of this model should be made. As in the model of Acemoglu and Zilibotti (1997) the development takes the form of the capital accumulation in the existing sector considered as domestic as well as opening and subsequent accumulation in a new sector in the economy considered as foreign operated. The extent of the development in the domestically operated sector is summarized by the capital stock $k_{1t}$. Likewise the extent of the development in the foreign operated sector is summarized by the capital stock $k_{2t}$ an initial value of which is lower than that of the domestic sector.

In addition to the equations (1), (2) and (3) the solution to the Program 1 must satisfy the following first order conditions:\(^2\):

$$1 = \beta E_t \left[ \frac{\partial g(i_{1t}, \theta_{t+1})}{\partial i_{1t}} \sum_{j=0}^{\infty} (\beta(1-\delta))^j f'(k_{1t+1+j}) \right], \quad (4)$$

$$1 = \beta E_t \left[ \frac{\partial g(i_{2t}, \theta_{t+1})}{\partial i_{2t}} \sum_{j=0}^{\infty} (\beta(1-\delta))^j F'(k_{2t+1+j}) \right], \quad (5)$$

$$u'(c_{1t}) = \lambda^{-1}. \quad (6)$$

The model discussed above is based on the assumption that the planner can perfectly enforce both parties to follow the plan. In the remaining of the paper, this assumption will be relaxed and a number of assumptions regarding incentive compatibility will be considered. These assumptions will essentially differ in the extent of the punishment the risk-averse agent would have to endure should he deviate from the plan.

### 2.2 Efficient growth mechanisms under limited commitment

We begin with the most stringent assumption on the punishment in case of violation of the contract. We will assume that in case of default the developing country will appropriate

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\(^1\)Similar assumptions appear e.g. in Marcet and Marimon (1992), and Jones and Manuelli (1990).

\(^2\)See Appendix A.1 for the derivation of the first order conditions.
the capital stock corresponding to the domestically operated sectors $k_{1t}$. The newly opened foreign sector will no longer be productive. This assumption can be justified on the grounds that the newly opened sector can be totally dependent on the technology transferred from the industrialized world. A similar assumption has been considered by Marcet and Marimon (1998) within the framework of a partnership with limited commitment.

Hence, the failure to honor the contract will result in closing down the sector which cannot be operated using domestically available technologies. In this case, the country will switch to autarky and will stay there forever. The problem the country would face in autarky will take the following form:

$$\max_{\{c_t,i_t\}_{t=0}^\infty} E_0 \left[ \sum_{t=0}^\infty \beta^t u(c_t) \right]$$

subject to

$$c_t + i_t = f(k_t),$$
$$k_{t+1} = (1 - \delta)k_t + g(i_t, \theta_{t+1}),$$

where $c_t \geq 0, i_t \geq 0$, and the initial values $k_0, \theta_0$ are given by the corresponding values of capital stock of the domestically operated sector and the shock value at the time of deviation. Using the arguments of standard dynamic programming one can show\(^3\) the existence of the time invariant policy functions $i(k, \theta), c(k, \theta)$ and a value function $V^a(k, \theta)$. Hence, the reservation value for the risk-averse agent at time $t$ is the utility of the autarkic solution $V^a(k_{1t}, \theta_t)$ given the capital stock $k_{1t}$ and the productivity shock $\theta_t$. The optimal allocations can be found by solving the following planner’s problem with $\lambda \in \mathbb{R}_+$ and the participation constraint imposed on agent 1:

**Program 2**

$$\max_{\{c_{1t},\tau_t,i_{1t},i_{2t}\}_{t=0}^\infty} E_0 \left[ \sum_{t=0}^\infty \beta^t \left[ \lambda u(c_{1t}) + (-\tau_t) \right] \right]$$

subject to

$$c_{1t} - \tau_t + i_{1t} + i_{2t} = f(k_{1t}) + F(k_{2t}),$$
$$k_{1t+1} = (1 - \delta)k_{1t} + g(i_{1t}, \theta_{t+1}),$$
$$k_{2t+1} = (1 - \delta)k_{2t} + g(i_{2t}, \theta_{t+1}),$$
$$E_t \left[ \sum_{i=0}^\infty \beta^i u(c_{1t+i}) \right] \geq V^a(k_{1t}, \theta_t),$$

with $c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0$ given.

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\(^3\)See appendix A.2 for the solution to the dynamic programming problem under the autarky.
Since the constraint (10) involves expected values of the future variables, Program 2 is not a special case of the standard dynamic programming problems, and the Bellman equation will not be satisfied. However, as shown by Marcet and Marimon (1998) this problem falls into a general class of problems, which can be cast into alternative recursive framework. The recursive saddle point problem associated with Program 2 will be given by

$$\max_{\{c_{1t}, \tau_{1t}, i_{1t}, i_{2t}\}} \min_{\mu_t} \mathcal{H} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (\lambda + M_{t-1}) u(c_{1t}) + (-\tau_t) + \mu_t \left( u(c_{1t}) - V^a(k_{1t}, \theta_t) \right) \right\}$$

subject to (7)-(9) and

$$M_t = M_{t-1} + \mu_t, \quad M_{-1} = 0,$$

$$\mu_t \geq 0.$$  

Indeed, the corresponding Lagrangian is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda u(c_{1t}) + (-\tau_t) + \mu_t \left( E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t}) \right] - V^a(k_{1t}, \theta_t) \right) \right\}$$

subject to (7)-(9), given $\mu_t \geq 0$, where $\beta^{-t} \mu_t$ is the Lagrange multiplier of (10) at $t$. The law of iterated expectations allows to imbed the conditional expectations $E_t$ into $E_0$. Furthermore, reordering the terms and introducing the law of motion for $M_t$ yields the above result.

As shown by Marcet and Marimon (1998), under certain assumptions$^4$ the solution to the recursive saddle point problem obeys a saddle point functional equation. Within our framework their result implies that there exists a unique value function,

$$W(k_1, k_2, M, \theta) = \min_{\mu \geq 0} \max_{\{c_1, \tau, i_1, i_2\}} \left\{ (\lambda + M) u(c_1) + (-\tau) + \mu \left( u(c_1) - V^a(k_1, \theta) \right) \right. \right.$$

$$\left. + \beta E \left[ W(k_1', k_2', M', \theta') \mid \theta \right] \right\}$$

subject to

$$c_1 - \tau + i_1 + i_2 = f(k_1) + F(k_2),$$

$$k_j' = (1 - \delta)k_j + g(i_j, \theta'), \quad \text{for } j = 1, 2,$$

$$M' = M + \mu,$$

$$c_1, i_1, i_2 \geq 0,$$

$^4$Marcet and Marimon (1998) state some interiority conditions needed for the existence of the saddle point problem. These are trivially satisfied in the framework studied here.
for all \((k_1, k_2, M, \theta)\) and such that \(W(k_{10}, k_{20}, M_{-1}, \theta_0)\) is the value of Program 2. The policy correspondence associated with the above saddle point functional equation is given by

\[
\psi(k_1, k_2, M, \theta) \in \operatorname{arg \, min}_{\mu \geq 0} \max_{\{c_1, \tau, i_1, i_2\}} \left\{ (\lambda + M) u(c_1) + (-\tau) + \mu (u(c_1) - V^a(k_1, \theta)) \right\} + \beta E \left[ W(k'_1, k'_2, M', \theta') \mid \theta \right]
\]

subject to (12) - (15).

The key results demonstrated by Marcet and Marimon (1998) ensures that the optimal solution of Program 2 satisfies \((c_{1t}, \tau_t, i_{1t}, i_{2t}, \mu_t) = \psi(k_{1t}, k_{2t}, M_{t-1}, \theta_t)\) for all \(t\) with the initial conditions \((k_{10}, k_{20}, 0, \theta_0)\). That is there exist a time invariant policy correspondence \(\psi\) such that only the values of a small number of past variables \((k_{1t}, k_{2t}, M_{t-1}, \theta_i)\) matter. Hence, the problem is now in a recursive framework the solution to which can now be obtained from studying the saddle point functional equation.

Denoting \(\gamma_{1t}\) and \(\gamma_{2t}\) the Lagrange multipliers of the constraints (8) and (9), the first order conditions for this problem become:

\[
(\lambda + M_t) u'(c_{1t}) = 1,
\]

\[
-1 - \beta E_t \left[ \gamma_{1t+1} \frac{\partial g(i_{1t}, \theta_{t+1})}{\partial i_{1t}} \right] = 0,
\]

\[
-1 - \beta E_t \left[ \gamma_{2t+1} \frac{\partial g(i_{2t}, \theta_{t+1})}{\partial i_{2t}} \right] = 0,
\]

\[
f'(k_{1t}) - \mu_t \frac{\partial V^a}{\partial k_{1t}}(k_{1t}, \theta_t) + \gamma_{1t} - \beta(1 - \delta)E_t \left[ \gamma_{1t+1} \right] = 0,
\]

\[
F'(k_{2t}) + \gamma_{2t} - \beta(1 - \delta)E_t \left[ \gamma_{2t+1} \right] = 0,
\]

\[
E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] - V^a(k_{1t}, \theta_t) \geq 0,
\]

\[
\mu_t \left[ E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] - V^a(k_{1t}, \theta_t) \right] = 0,
\]

in addition to the technological constraints (7)-(9), the law of motion (11) for the co-state variable \(M_t\), and non-negativity of the Lagrange multiplier \(\mu_t \geq 0\).
3 Solutions to the growth models

In this section we will present the numerical solutions for various models of this paper as well as describe the algorithms for obtaining them. To obtain the numerical solution to the models we will rely on the parameterized expectation approach. With some exceptions, the functional forms utilized here are similar to those of Marcet and Marimon (1992). These are

\[ f(k_{1t}) = A^\alpha k_{1t}^\alpha \quad \text{and} \quad F(k_{2t}) = \tilde{A}k_{2t}^\alpha, \]  

(23)

\[ g(i_t, \theta_{t+1}) = a(\theta_{t+1} + s) \frac{i_t}{(1 + i_t)} + b, \]  

(24)

\[ u(c_{it}) = c_{it}^{\gamma+1}/(\gamma + 1), \]  

(25)

\[ \log \theta_t = \rho \log \theta_{t-1} + \epsilon_t, \]  

(26)

where \( \{\epsilon_t\} \) are independent normally distributed random variables with zero mean and variance \( \sigma^2_\epsilon \).

3.1 Solving the problem with full enforcement

With the chosen functional forms the optimality conditions for the case of full enforcement are the following:

\[ (1 + i_{1t})^2 = \beta E_t \left[ a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1 - \delta))^j A\alpha(k_{1t+1+j})^{\alpha-1} \right], \]  

(27)

\[ (1 + i_{2t})^2 = \beta E_t \left[ a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1 - \delta))^j \tilde{A}\alpha(k_{2t+1+j})^{\alpha-1} \right], \]  

(28)

\[ c_{1t}^\gamma = \lambda^{-1}, \]  

(29)

\[ c_{1t} - \tau_t + i_{1t} + i_{2t} = A^\alpha k_{1t} + \tilde{A}k_{2t}^\alpha, \]  

(30)

\[ k_{1t+1} = (1 - \delta) k_{1t} + a(\theta_{t+1} + s) \frac{i_{1t}}{(1 + i_{1t})} + b, \]  

(31)

\[ k_{2t+1} = (1 - \delta) k_{2t} + a(\theta_{t+1} + s) \frac{i_{2t}}{(1 + i_{2t})} + b. \]  

(32)

The first step of the PEA is to substitute the conditional expectations in (27) and (28) by the flexible functional forms that depend on the state variables and some coefficients\(^5\). Each

\(^5\)see Marcet and Lorenzoni (1998) for further details on the implementation of PEA.
of the parameterized expectations $i = 1, 2$ takes the form:

$$\psi(\omega^i; k_{1t}(\omega), k_{2t}(\omega), \theta_t) = \exp(\omega_1^i + \omega_2^i \log k_{1t}(\omega) + \omega_3^i \log k_{2t}(\omega) + \omega_4^i \log \theta_t),$$

where $\omega = (\omega^1; \omega^2)$. The use of the exponential polynomial guarantees that the left hand side of (27) and (28) would be positive. Increasing the degree of the polynomial would allow to approximate the solution with arbitrary accuracy\(^6\).

The algorithm for solving the model takes the following steps:

- (Step I) Fix the initial conditions and draw a series of $\{\theta_t\}_{t=1}^T$ that obeys (26) with $T$ sufficiently large.

- (Step II) For a given $\omega$ substitute the conditional expectations in (27) and (28) to yield:

$$(1 + i_{it})^2 = \delta \psi(\omega^i; k_{1t}(\omega), k_{2t}(\omega), \theta_t) \text{ for } i = 1, 2 \tag{33}$$

- (Step III) Using the realizations of $\theta_t$ obtain recursively from (33) and (29)-(32) a series of the endogenous variables $\{c_{1t}(\omega), \tau_t(\omega), i_{1t}(\omega), i_{2t}(\omega), k_{1t}(\omega), k_{2t}(\omega)\}$ for this particular $\omega$.

- (Step IV) The next step involves running two separate non-linear regressions. The role of the dependent variables will be performed by the expressions inside the conditional expectation in the RHS of (27) and (28). Namely, the 'dependent variables' $Y_{1t}(\omega)$ and $Y_{2t}(\omega)$ would take form

$$Y_{1t}(\omega) \equiv a(\theta_{t+1} + s) \sum_{j=0}^\infty (\beta(1 - \delta))^j A\alpha(k_{1t+1+j}(\omega))^{\alpha - 1},$$

$$Y_{2t}(\omega) \equiv a(\theta_{t+1} + s) \sum_{j=0}^\infty (\beta(1 - \delta))^j \tilde{A}\alpha(k_{2t+1+j}(\omega))^{\alpha - 1}.$$

Now, letting $S'(\omega)$ be the result of the following regression:

$$Y_{it}(\omega) = \exp(\xi_1^i + \xi_2^i \log k_{1t}(\omega) + \xi_3^i \log k_{2t}(\omega) + \xi_4^i \log \theta_t) + \eta_{it},$$

for $i = 1, 2$, define $S(\omega) \equiv (S^1(\omega), S^2(\omega))$.

\(^6\)The fact that PEA can provide arbitrary accuracy if the approximation function is refined and a proof of convergence to the correct solution are given in Marcet and Marshall (1994). In practice the choice of degree of the exponential polynomial can be guided by the test for accuracy in simulation proposed by den Haan and Marcet (1994). Some practical issues on dealing with higher-order polynomials in the approximation function are discussed in den Haan and Marcet (1990).
(Step V) The final step involves using an iterative algorithm to find the fixed point of $S$, and the set of coefficients $\omega_f = S(\omega_f)$ which would give the solution for the endogenous variables $\{c_{1t}(\omega_f), \tau_t(\omega_f), i_{1t}(\omega_f), i_{2t}(\omega_f), k_{1t}(\omega_f), k_{2t}(\omega_f)\}$.

### 3.2 Solving the problem with limited commitment

This section shows how to solve the model with limited enforcement using PEA adapted from Marcet and Marimon (1992). The main difference from the algorithm discussed above is that here the participation constraint might be binding in some periods and slack in the others. Furthermore, there is one more expectation to parameterize and an additional (co-)state variable $M_{t-1}$ to include into the parameterization.

The following optimality conditions are to be satisfied:

$$
\mu_t \left[ u(c_{1t}) + E_t \left[ \sum_{i=1}^{\infty} \beta^i u(c_{1t+i}) \right] - V^a(k_{1t}, \theta_t) \right] = 0,
$$

$$
E_t \left[ \sum_{i=1}^{\infty} \beta^i u(c_{1t+i}) \right] - V^a(k_{1t}, \theta_t) \geq 0,
$$

$$
c^\gamma_{1t} = 1/(\lambda + \mu_t + M_{t-1}),
$$

$$
M_t = M_{t-1} + \mu_t,
$$

$$
c_{1t} - \tau_t + i_{1t} + i_{2t} = Ak_{1t}^\alpha + \tilde{A}k_{2t}^\alpha,
$$

$$
k_{1t+1} = (1 - \delta)k_{1t} + a(\theta_{t+1} + s)i_{1t}/(1 + i_{1t}) + b,
$$

$$
k_{2t+1} = (1 - \delta)k_{2t} + a(\theta_{t+1} + s)i_{2t}/(1 + i_{2t}) + b,
$$

$$
(1 + i_{1t})^2 = \beta E_t \left[ a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1 - \delta))^j
\times \left( A^\alpha(k_{1t+1+j})^{\alpha-1} - \mu_{t+j+1} \frac{\partial V^a(k_{1t+j+1}, \theta_{t+j+1})}{\partial k_{1t+j+1}} \right) \right],
$$

$$
(1 + i_{2t})^2 = \beta E_t \left[ a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1 - \delta))^j \tilde{A}^\alpha(k_{2t+1+j})^{\alpha-1} \right],
$$

in addition to the inequality constraint $\mu_t \geq 0$ and the initial conditions$^7$.

In order to solve this model with PEA the algorithm described for the case of full enforce-

$^7$From (19) and (20) using recursive substitution and the law of iterated expectations yields the following
ment should be modified in the following way. First, in step II parameterize the conditional expectations in (34), (41) and (42) to yield

\[(1 + i_t(\omega))^2 = \delta \psi(\omega^3; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) \text{ for } i = 1, 2, \quad (43)\]

\[
\mu_t \left[ u(c_{1t}(\omega)) + \beta \psi(\omega^3; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) - V^a(k_{1t}(\omega), \theta_t) \right] = 0,
\]

where \(\omega = (\omega^1, \omega^2, \omega^3)\).

In step III the participation constraint should be taken into account. One way to proceed is to initially assume that the participation constraint is not binding, then \(\mu_t(\omega) = 0, M_t(\omega) = M_{t-1}(\omega)\), and the solution for \(c_{1t}(\omega)\) follows from (36). For this solution one has to check whether the constraint is indeed satisfied, that is if

\[u(c_{1t}(\omega)) + \beta \psi(\omega^3; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) \geq V^a(k_{1t}(\omega), \theta_t).\]

If that is the case one can proceed by solving for the rest of the endogenous variables from (43) and the feasibility constraints (38) - (40). Otherwise, the participation constraint must be binding, that is

\[u(c_{1t}(\omega)) + \beta \psi(\omega^3; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) = V^a(k_{1t}(\omega), \theta_t),\]

from which the solution for \(c_{1t}(\omega)\) follows. The value of the multiplier \(\mu_t(\omega)\) then follows from (36), the value of \(M_t(\omega)\) from the law of motion (37), and the rest of the endogenous variables from (43) and (38) - (40).

Now, step IV will involve running three non-linear regressions for \(i = 1, 2, 3\) of the form

\[Y_{it}(\omega) = \exp(\xi^i_1 + \xi^i_2 \log k_{1t}(\omega) + \xi^i_3 \log k_{2t}(\omega) + \xi^i_4 \log \theta_t + \xi^i_5 M_{t-1}(\omega)) + \eta_{it},\]

expressions for the lagrange multipliers \(\gamma_{1t}\) and \(\gamma_{2t}\)

\[
\gamma_{1t} = \beta E_t \left[ a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta (1 - \delta))^j \left( \frac{\partial V^a}{\partial k_{1t+j}}(k_{1t+j}, \theta_{t+j}) - f'(k_{1t+j}) \right) \right],
\]

\[
\gamma_{2t} = -\beta E_t \left[ a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta (1 - \delta))^j F'(k_{2t+j}) \right].
\]

Substituting the the above expressions into (17) and (18) respectively, using again the law of iterated expectations and the functional forms for the production and investment functions yields the optimality conditions (41) and (42).
where the ‘dependent variables’ are given by

\[
Y_{1t}(\omega) \equiv a (\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j \left[ A_0(k_{1t+1+j}(\omega))^{\alpha-1} - \mu_{t+j+1}(\omega) \frac{\partial V^a(k_{1t+j+1}(\omega), \theta_{t+j+1})}{\partial k_{1t+j+1}} \right],
\]

\[
Y_{2t}(\omega) \equiv a (\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j \tilde{A}_0(k_{2t+1+j}(\omega))^{\alpha-1},
\]

\[
Y_{3t}(\omega) \equiv \sum_{i=1}^{\infty} \beta^i u(c_{1t+i}(\omega)).
\]

The last step is similar to the one in the case of full enforcement.

A few notes on the algorithm should be made. First, in this algorithm \( \mu_t \) will be positive by construction. Second, step IV involves calculation of the derivative of the value function in the autarky with respect to its first argument. Marcet and Marimon (1992) provide derivation of this derivative which is convenient for computational purposes. The computation algorithm is given in Appendix A.2.

### 3.3 Numerical solutions to the models

In this section we present the simulated series for the models discussed above. First, a short note should be made on the parameterization of the model. The values of the parameters used in the simulations except for the productivity parameters \( A \) and \( \tilde{A} \) are similar to those of Marcet and Marimon (1992). This concerns all the models considered throughout the paper. The choice of values for the depreciation rate of the capital (\( \delta \)) and the discount factor (\( \beta \)) allows to interpret one period as a year. The values of the parameters are summarized in Table 1.

[insert Table 1 about here.]

A note on the weight \( \lambda \) in the planner’s problem should be made. In all the reported simulations the value of \( \lambda \) is set to make expected discounted transfers at \( t = 0 \) equal to zero. This would ensure that the series reported corresponds to the equilibrium contract.

[insert Figure 1 about here.]

The simulation results for the environment with full enforcement are presented in Figure 1. These results will be compared with those obtained in the autarkic environment (see Figure
The initial value of capital stock in the domestic sector is set to one, while the foreign operated sector is initially assumed to be nonexistent.\footnote{This assumption is made to make Autarky directly comparable with other environments. In addition, as in Marcet and Marimon (1992) we assume that the initial capital stock in the Autarkic environment equals to one.}

The results can be summarized in the following way. First, as expected, the consumption of the risk-averse agent in the PO environment is constant both in the steady state and along the transition. All the risk is born by the risk neutral agent, which is also reflected in the volatility of the transfers in the steady state.

Second, under full enforcement the developing country borrows heavily during the initial periods in order to boost investment in both sectors of the economy. Due to the access to external financing, the mean growth rate of output raises from 2.4% to 8.4% during the first 15 periods, and from 1.4% to 3.8% during the first 35 periods.

Third, during the initial periods the investment rates under PO environment are significantly higher that those in the autarky. Under full enforcement, as the capital accumulates in both sectors the investment rates decline. The opposite is observed in the autarkic environment. Higher investment level in the foreign sector than that of the domestic is due to the lower initial capital stock in the former. Remarkably, in the steady state the investment rates under PO environment are more volatile than those in the autarky.

Finally, under full enforcement access to external financial opportunities results in a welfare gain equivalent to a 92% "increase in consumption". By "increase in consumption" we refer to a permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other environments.

Remarkably, all of the results reported for the PO environment are also applicable to the model with limited commitment corresponding to Program 2. The implication of this finding is that technological gains from external financing opportunities may eliminate the default risk.

A comment should be made on this finding according to which the solutions to the case of full enforcement and limited enforcement coincide. The fact that participation constraint turns out to be never binding can driven by the assumption of the punishment in case of deviation from the optimal plan, which is extremely severe. Should the country default it will lose not only the technological advantage and capital accumulated in the newly opened sector but also a possibility to develop this sector on its own. In the remaining of the paper we will address the issue of default punishment which might give some qualitatively different results.
4 A Model with Two-sector Autarky

In this section, we will modify the assumption concerning the punishment incurred by the developing country in case of deviation from the optimal plan. It will be assumed that failure to follow the plan would result in the loss of the technological advantage in the newly opened sector\(^9\). However, the newly open sector will remain productive with the productivity level of the domestically operated sector. Furthermore, the country will preserve the accumulated capital in both sectors. In this formulation, the autarky would be given by

**Program 3**

\[
\max_{\{c_t, i_{1t}, i_{2t}\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]
\]

subject to

\[
c_{1t} + i_{1t} + i_{2t} = f(k_{1t}) + f(k_{2t}), \quad (44)
\]

\[
k_{jt+1} = (1 - \delta)k_{jt} + g(i_{jt}, \theta_{t+1}), \text{ for } j = 1, 2 \quad (45)
\]

with \(c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0\) given.

The arguments from the standard dynamic programming will ensure the existence of the time invariant policy functions \(i_1(k_1, k_2, \theta), i_2(k_1, k_2, \theta), c(k_1, k_2, \theta)\) and a value function \(V^{a2}(k_1, k_2, \theta)\). Hence, the reservation value for the agent 1 at time \(t\) is the utility of the autarkic solution \(V^{a2}(k_{1t}, k_{2t}, \theta_t)\) given the capital stock accumulated in the domestically operated sector \(k_{1t}\), the capital stock of the newly opened sector \(k_{2t}\) and the productivity shock \(\theta_t\).\(^{10}\)

Under these less stringent assumptions on the default punishment, the optimal allocations can be found by solving the following planner’s problem with \(\lambda \in \mathbb{R}_+\) and the participation constraint imposed on agent 1.

**Program 4**

\[
\max_{\{c_{1t}, \tau_t, i_{1t}, i_{2t}, k_{1t}, k_{2t}\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \lambda u(c_{1t}) + (-\tau_t) \right] \right]
\]

subject to

\[
c_{1t} - \tau_t + i_{1t} + i_{2t} = f(k_{1t}) + F(k_{2t}), \quad (46)
\]

\[
k_{jt+1} = (1 - \delta)k_{jt} + g(i_{jt}, \theta_{t+1}), \text{ for } j = 1, 2 \quad (47)
\]

\(^9\)This assumption is close in spirit to those of Cohen and Sachs (1986) or Eaton and Gersovitz (1984) where foreign debt repudiation results in permanent loss of productive efficiency.

\(^{10}\)See Appendix A.3 for the optimality conditions corresponding to Program 3.
\[
E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] \geq V^{a2}(k_{1t}, k_{2t}, \theta_t), \tag{48}
\]

with \( c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0 \) given.

Once again, in the above framework, the steady state distributions of capital will differ under full and limited enforcement due to the technology transfers. This feature would distinguish the present setup from the framework of Marcet and Marimon (1992) as far as the growth incentives for integration are concerned.

Similar to Program 2, the present problem can be cast into recursive framework the solution to which will be obtained from studying the saddle point functional equation. Denoting \( \gamma_{1t} \) and \( \gamma_{2t} \) the Lagrange multipliers of the constraints (47), the first order conditions for this problem become:

\[
(\lambda + M_t) u'(c_{1t}) = 1,
\]
\[
-1 - \beta E_t \left[ \gamma_{1t+1} \frac{\partial g(i_{1t}, \theta_{t+1})}{\partial i_{1t}} \right] = 0,
\]
\[
-1 - \beta E_t \left[ \gamma_{2t+1} \frac{\partial g(i_{2t}, \theta_{t+1})}{\partial i_{2t}} \right] = 0,
\]
\[
f'(k_{1t}) - \mu_t \frac{\partial V^{a2}}{\partial k_{1t}}(k_{1t}, k_{2t}, \theta_t) + \gamma_{1t} - \beta(1 - \delta) E_t \left[ \gamma_{1t+1} \right] = 0,
\]
\[
F'(k_{2t}) - \mu_t \frac{\partial V^{a2}}{\partial k_{2t}}(k_{1t}, k_{2t}, \theta_t) + \gamma_{2t} - \beta(1 - \delta) E_t \left[ \gamma_{2t+1} \right] = 0,
\]
\[
E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] - V^{a2}(k_{1t}, k_{2t}, \theta_t) \geq 0,
\]
\[
\mu_t \left[ E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] - V^{a2}(k_{1t}, k_{2t}, \theta_t) \right] = 0,
\]
in addition to the technological constraints (46)-(48), the law of motion for the co-state variable \( M_t \),
\[
M_t = M_{t-1} + \mu_t, \quad M_{-1} = 0
\]
and non-negativity of the Lagrange multiplier \( \mu_t \geq 0 \).

Substituting the chosen functional forms and simplifying the first order conditions in a manner similar to the one described in footnote 6 yields the following optimality conditions:

\[
\mu_t \left[ u(c_{1t}) + E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] - V^{a2}(k_{1t}, k_{2t}, \theta_t) \right] = 0,
\]
\[ E_t \left[ \sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] - V^a_t(k_{1t}, k_{2t}, \theta_t) \geq 0, \]
\[ c_{1t}^* = 1/(\lambda + \mu_t + M_{t-1}), \]
\[ M_t = M_{t-1} + \mu_t, \]
\[ c_{1t} - \tau_t + i_{1t} + i_{2t} = A k_{1t}^a + \tilde{A} k_{2t}^a, \]
\[ k_{jt+1} = (1 - \delta) k_{jt} + a(\theta_{t+1} + s) i_{jt}/(1 + i_{jt}) + b, \text{ for } j = 1, 2 \]
\[ (1 + i_{1t})^2 = \beta E_t \left[ a \left( \theta_{t+1} + s \right) \sum_{j=0}^{\infty} \beta^j (1 - \delta)^j \right. \]
\[ \times \left( A\alpha(k_{t+1} + j)^{\alpha-1} - \mu_{t+j+1} \frac{\partial V^a_t(k_{1t+j+1}, k_{2t+j+1}, \theta_{t+j+1})}{\partial k_{1t+j+1}} \right), \]
\[ (1 + i_{2t})^2 = \beta E_t \left[ a \left( \theta_{t+1} + s \right) \sum_{j=0}^{\infty} \beta^j (1 - \delta)^j \right. \]
\[ \times \left( \tilde{A}\alpha(k_{t+1} + j)^{\alpha-1} - \mu_{t+j+1} \frac{\partial V^a_t(k_{1t+j+1}, k_{2t+j+1}, \theta_{t+j+1})}{\partial k_{2t+j+1}} \right), \]
in addition to non-negativity of the Lagrange multiplier \( \mu_t \geq 0 \) and the initial conditions.

5 Characterization of equilibria

We solve the model in Program 4 with the PEA using an algorithm similar to the one described for the model in Program 2. As before, in all simulations the TFP parameter of the domestic sector \( (A) \) was set to one. When it comes to the TFP parameter of the foreign operated sector \( (\tilde{A}) \), we consider three representative cases which differ in the magnitude of the technological transfers.

The simulation results are summarized in Figures 3-5 and Tables 3-5. We compare three institutional environments: the autarky equilibrium corresponding to Program 3 denoted as "au" in Figures 3-5, Pareto optimum allocation with perfect enforcement denoted as "po", and the equilibrium with limited enforcement corresponding to Program 4 denoted as "pc". For these figures we plot the first 50 periods as representative of the transition from the low level of capital to the steady state, and periods 100 to 200 as representative of the steady state distribution.
5.1 Equilibria with no technological transfers

First, we consider the case with no technological transfers whatsoever, which in terms of TFP’s corresponds to $\bar{A} = A = 1$. This case is similar in structure to the growth models with full information of Marcet and Marimon (1992). Under lack of commitment, the behavior of the developing country is affected by the two opposing forces. On one hand, the country wants to default on its debt, something which would imply switching to autarky and staying there forever. Unlike the autarky assumption of the Program 2, Program 4 implies that the country would still be in a position to develop the foreign sector on its own with the expropriated capital to begin with. The opposing force is the threat of the punishment for defaulting. In this case, it is the loss of possibility to borrow in order either enhance growth or to smooth consumption against the unforeseen shocks or along the growth path. As before, the characterization of the capital accumulation and transfers during the transition can be obtained only from the numerical solutions, which are summarized in Figure 3 and Table 3.

[insert Figure 3 and Table 3 about here.]

An important feature of this case is that the steady state distributions of capital are quite similar across all the three environments, in both sectors. They are actually identical in the PC and PO environments as are the distributions of the corresponding investment rates. As reported in Table 3, the steady state capital stock in the autarky environment is slightly higher on average than in the other environments in either of the sectors. The reason for that is that in autarky the country has to self-insure against the cyclical fluctuations of output and the only source of self-insurance is the capital.

In each of the sectors, the investment is more volatile under full enforcement than under the autarky. This feature is similar to the one reported by Marcet and Marimon (1992), and represents an example where an increase in volatility of investment is desirable.

Despite absence of any technological spillovers, the positive effect of the access to external financing on growth is rather substantial under full enforcement. The growth rates go from 2.5 to 3% during the first 15 periods. Yet, this effect practically disappears once the assumption of perfect enforceability of contracts is relaxed. The overall gains, measured as permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other regimes, differ significantly in the PO and PC environments. Failure to perfectly enforce contracts reduces the welfare gains by the factor of 25. In fact, during the transition the consumption paths under autarky and under limited enforcement are very similar. As can be seen from Figure 3, the key difference is that the consumption series under PC is smoother than that under the autarky during the transition.
Furthermore, it is outright flat in the steady state while the consumption under autarky keeps fluctuating even in the steady state. Hence, with no technological transfers, the access to the external financing under limited enforcement allows to smooth out variation of output but not keep constant consumption along the transition. The possibility to smooth consumption through external financing results in the minor welfare gain under limited commitment. As in Marcet and Marimon (1992) enforcement constrains result in negligible transfers and severely reduce growth opportunities.

5.2 Equilibria with technological transfers of medium magnitude

The case with the technological transfers of medium magnitude is defined by two characteristic features. First, in the environments which grant access to the external financing, the foreign operated sector is more productive than the domestic one\(^\text{11}\). Second, the productivity differences between sectors are low enough to guarantee that the participation constraint is binding in some periods. The key feature of this case is that the productivity benefits introduce a gap between the steady state capital levels in the economy with and without external financing. The latter feature makes the punishment for default more severe than in the previous case but not severe enough to eliminate risk of default. The characteristics of the efficient accumulation mechanisms under the three considered institutional setups are summarized in Figure 4 and Table 4.

The simulations demonstrate several distinctive features of the setup which encompasses both productivity benefits from external financing and risk of default. These can be summarized in the following way:

First, despite the presence of the default risk in the environment with limited commitment the capital movements from and to the developing country are no longer negligible. This result distinguishes the present setup from both the equilibrium with no technological transfers discussed in the previous section and the framework of Marcet and Marimon (1992). This feature allows to conclude that presence of the default risk is not inconsistent with the capital flows of substantial magnitude.

Second, under limited enforcement the developing country borrows not only in order to smooth cyclical variation in consumption but also in order to invest heavier during the transition and hence foster growth. Remarkably, the borrower boosts investment in all productive

\(^{11}\)In terms of sectoral TFPs the case reported here corresponds to \(A = 1\) and \(\tilde{A} = 1.1\).
sectors and not only those affected by the technological transfers. Once again, in this prediction the current case differs from the case with no technological diffusion, be it two-sector model discussed above or one-sector framework of Marcet and Marimon (1992). That is, borrowing with an objective to promote growth can be an equilibrium outcome even in the environment with present risk of default.

Third, the behavior of the consumption path under limited enforcement is rather peculiar. During a few initial periods, the consumption path is flat. Although it is still lower that the consumption level under full enforcement, the series is well above the autarky consumption. That is, in this environment consumption smoothing along the growth path is no longer absent. As the capital accumulates, the participation constraint starts binding at certain period. After that the consumption in the limited commitment environment rises every time the incentive compatibility constraint binds. As in the case with no technological transfers, the shape of the consumption series reminds that of the autarky. However, during the all the transition periods there is a diminishing wedge between the two series. This can be attributed to the diminishing difference in the accumulated capital stock in the environments with full and limited enforcement. As in the case with no technological diffusion, under limited enforcement the steady state distribution is characterized by a flat consumption schedule which can lie either above or below the autarky path.

Since the default risk is still present during the transition, under limited enforcement the paths of investment, transfers, and capital stock differ from those in the Pareto optimum. Transfers from abroad to the developing country are lower in this case relative to the full enforcement outcome. The investment rates inherit the same feature. In fact, in the sector unaffected by the productivity benefits the investment series falls rather quickly to the autarky level. However, due to the heavy investment during the initial periods, the capital stock under limited enforcement stays above the autarky capital stock during the transition. The latter result holds for all sectors including the domestic one.

Another regularity concerns the overall capital stock of the economy in the steady state. As shown by Marcet and Marimon (1992) the capital stock of a country in an environment with limited commitment is lower than that in the autarky. The driving force behind this result is the need to use capital as the only means of self-insurance in the autarkic environment. The similar result is obtained in our framework in the case with no technological diffusion. When the technological transfers are present, however, this conclusion may no longer be true. Since the productivity of the foreign operated sector is higher under limited enforcement than in the autarky, so is the capital stock in the foreign sector. Hence, whether the overall capital stock will be higher in the autarky than under limited commitment depends on which of the two forces dominates. For instance, in the case with transfers of medium magnitude reported
in Table 4, under limited enforcement the capital stock in the domestic sector is lower than that in the autarky. The converse is true for the foreign operated sector.

Some characteristic features of the solutions following from our framework are in line with a number of documented empirical regularities. For instance, Marcet and Marimon (1992) state that the observed cross-country differences in borrowing patterns and rich structure of capital flows find little explanation in the models of sustained growth. On the contrary, our framework predicts that under limited commitment, the extent to which a developing country will borrow depends on the magnitude of productivity gains associated with external financing relative to the productivity in the autarky.

Another regularity is reported by Getler and Rogoff (1990) who document that the level of foreign debt in the developing countries is positively correlated with their GNP. This observation is in line with the predictions of our model as well. Indeed, countries which highly benefit from technological transfers in the foreign operated sector will be able not only to increase production due to the productivity gains but also due to the higher capital stock in all sectors. The latter stems from increased investment levels financed through transfers from abroad. Such countries will tend to have both higher income level and higher level of foreign debt.

### 5.3 Equilibria with technological transfer of high magnitude

When the magnitude of technological transfers is high enough the punishment for default becomes so severe that the participation constraint turns out to be never binding. Hence, the solution under limited commitment and that under perfect enforcement will coincide. This compels us to reiterate the conclusion obtained earlier from the model with one-sector autarky. Our results suggest that presence of technological benefits associated with external financing may eliminate risk of default. The latter is true even though these benefits are enjoyed only by some sectors of the developing economy. The simulation results for the case with technological transfer of high magnitude are presented in Figure 5 and Table 5.

One final note will be made concerning the relation between the productivity benefits and the corresponding welfare gains. In the reported example the TFP level in the foreign operated sector \( (\bar{A}) \) is set to 1.35. This particular choice is motivated by the desire to find the lowest level of \( \bar{A} \), which would ensure that the participation constraint does not bind. In this case, the welfare gain, measured as a permanent increase in consumption that would equate the present value of utility under the autarky with the present values achieved in the other environments,
is large. It corresponds to the increase in consumption of 26%. Notice that these gains are driven by two forces. On one hand, it is higher productivity of the foreign operated sectors under PC than that under autarky which takes the credit. On the other hand, the spillovers increase the default punishment and by that facilitate borrowing during the initial periods in order to foster growth. The importance of the latter force for welfare improvement is more obvious in the case with no transfers reported in Table 3. In the absence of technological diffusion, the failure to enforce contracts results in a welfare loss corresponding to change in consumption of 3.4%. With introduction of moderate technological transfers, corresponding to the TFP level in the foreign operated sector ($\tilde{A}$) of 1.1, the difference between welfare gains under full and limited enforcement falls by more than a half and becomes 1.6%. This reduction of relative welfare benefits can be attributed to an increase in the punishment for default.

To summarize, even moderate technological benefits substantially reduce the negative effect on welfare of the failure to perfectly enforce the contracts. That is, in our framework technological transfers play a role of an important enforcement mechanism.

6 Conclusion

The main objective of this study was to enrich the framework of Marcet and Marimon (1992), which considers the effects of alternative financing opportunities on economic growth under lack of commitment. The key novelty is that the access to the external financing is assumed to result in technological transfers to the developing countries from the rest of the world. We study a two-sector growth model in the environments which differ to the extent the borrowing contracts with the rest of the world are being enforced. Furthermore, we examine different assumptions concerning the default punishment and their implications for growth, welfare and borrowing patterns. The principal conclusions of this paper can be summarized in the following way:

First, our model suggests that technological transfers to a developing country from the rest of the world may eliminate risk of default even though they affect only some sectors of the economy.

Second, we conclude that the existence of substantial capital flows from the developed to developing countries is not inconsistent with the presence of the default risk. In this respect predictions of our model distinguish themselves from those of Marcet and Marimon (1992).

Third, we overcome the difficulty that the models of sustained growth have in explaining the rich structure of observed capital flows and the "wide spectrum of borrowing patterns across low- and middle-income countries" (Marcet and Marimon, 1992:221). Our framework
predicts that under limited commitment the pattern of capital flows depends heavily on the productivity benefits associated with the external financing opportunities.

Forth, in our framework even moderate technological benefits associated with external financing opportunities may substantially reduce the negative effect on the welfare of the failure to perfectly enforce contracts. In this respect, we conclude that technological transfers may play a role of an important enforcement mechanism.

Finally, we show that absence of technological diffusion in an environment with limited commitment may result in scarce capital flows to less developed countries and substantially reduce their growth opportunities. On the over hand, presence of technological transfers in this environment will induce a developing country to use foreign capital to both smooth consumption against unforeseen shocks as well as along the growth path. Moreover, along the transition the foreign capital will be used to invest more heavily in all the sectors of the economy including those unaffected by the spillovers. The latter will result in faster growth as well as more substantial welfare gains.
References


A Appendix

A.1 Derivation of the first order conditions in (4), (5) and (6).

Using the arguments of standard dynamic programming (see Lucas and Stockey, 1989) one can show the existence of the time invariant policy functions $i_1(k_1, k_2, \theta)$, $i_2(k_1, k_2, \theta)$ and a value function $V(k_1, k_2, \theta)$. The Lagrangian for the problem is given by

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \lambda u(c_{1t}) + (-\tau_t) - \lambda_{1t} \left( c_{1t} - \tau_t + i_{1t} + i_{2t} - f(k_{1t}) - F(k_{2t}) \right) - \mu_{1t}(k_{1t} - (1-\delta)k_{1t-1} - g(i_{1t-1}, \theta_t)) - \mu_{2t}(k_{2t} - (1-\delta)k_{2t-1} - g(i_{2t-1}, \theta_t)) \right],$$

where $\lambda_{1t}, \mu_{1t}$ and $\mu_{2t}$ are the Lagrange multipliers associated with the constraints (1), (2) and (3). The corresponding f.o.c. are given by

$$\lambda u'(c_{1t}) = \lambda_{1t}, \quad (49)$$
$$1 = \lambda_{1t}, \quad (50)$$
$$-\lambda_{1t} + \beta E_t \left[ \mu_{1t+1} \frac{\partial g(i_{1t}, \theta_{t+1})}{\partial i_{1t}} \right] = 0, \quad (51)$$
$$-\lambda_{1t} + \beta E_t \left[ \mu_{2t+1} \frac{\partial g(i_{2t}, \theta_{t+1})}{\partial i_{2t}} \right] = 0, \quad (52)$$
$$\lambda_{1t} f'(k_{1t}) - \mu_{1t} + (1-\delta)\beta E_t \left[ \mu_{1t+1} \right] = 0, \quad (53)$$
$$\lambda_{1t} F'(k_{2t}) - \mu_{2t} + (1-\delta)\beta E_t \left[ \mu_{2t+1} \right] = 0. \quad (54)$$

From the equation (53) using recursive substitution yields

$$\mu_{1t} = E_t \left[ \sum_{j=0}^{\infty} (\beta(1-\delta))^j f'(k_{1t+j}) \lambda_{1t+j} \right].$$

Substituting the latter into (51) and using (50) as well as the law of iterated expectations yields

$$1 = \beta E_t \left[ \frac{\partial g(i_{1t}, \theta_{t+1})}{\partial i_{1t}} \sum_{j=0}^{\infty} (\beta(1-\delta))^j f'(k_{1t+1+j}) \right].$$

The condition (5) is derived using the similar argument from (54),(52), and (50). The condition (6) follows directly from (49) and (50).
A.2 Approximating the value function and its derivative in the one-sector autarky.

The Bellman equation corresponding to the one-sector autarky is given by

\[ V^a(k; \theta) = \max_{(c, i) \in A(k)} \left\{ u(c) + \beta E[V^a(k', \theta') \mid \theta] \right\}, \]

subject to

\[ A(k) = \{(c, i) \in \mathbb{R}_+^2 : c + i = f(k)\}, \]
\[ k' = (1 - \delta)k + g(i, \theta'). \]

Denoting by \( V'(k; \theta) \) the derivative of the value function with the respect to its first argument, the first order condition for the problem becomes

\[ u'(c) = \beta E \left[ V'^a(k', \theta') \frac{\partial g(i, \theta')}{\partial i} \mid \theta \right]. \] (55)

Applying the theorem of Benveniste - Scheinkman\(^\text{12}\) yields the following condition for the derivative:

\[ V'^a(k, \theta) = u'(c)f'(k) + \beta(1 - \delta)E[V'^a(k', \theta') \mid \theta]. \]

Rewriting the latter in the sequence form, using recursive substitution and the law of iterated expectations yields

\[ V'^a(k_t, \theta_t) = E_t \left[ \sum_{j=0}^{\infty} (\beta(1 - \delta))^j u'(c_{t+j})f'(k_{t+j}) \right]. \] (56)

Now, rewriting (55) in the sequence form, using (56) and the law of iterated expectations yields the first order condition for the autarky

\[ u'(c_t) = \beta E_t \left[ \frac{\partial g(i_{1t}, \theta_{1t+1})}{\partial i_{1t}} \sum_{j=0}^{\infty} (\beta(1 - \delta))^j u'(c_{t+j+1})f'(k_{1t+1+j}) \right]. \] (57)

In order to approximate the value function and its derivative the following algorithm can be used. First, parameterize the conditional expectation in (56) as

\[ \psi(\omega, k_t, \theta_t) = \exp(P_n(\log(k_t), \log(\theta_t))), \]

\(^{12}\)see Lucas and Stockey (1989) or Marcet and Marimon (1992) for details.
where $P_n$ is a polynomial of degree $n$. Then, run a non-linear regression, which for $n = 2$ takes the form:

$$Y_t = \exp(\omega_1 + \omega_2 \log(k_t) + \omega_3 \log(\theta_t) + \omega_4 (\log(k_t))^2 + \omega_5 \log(k_t) \log(\theta_t) + \omega_6 (\log(\theta_t))^2) + \eta_t,$$

where the dependent variable $Y_t$ is given by the expression inside the conditional expectation in (56) evaluated the the autarky solution $\{c_t, k_t\}_t^\infty$.

A similar approach can be used to approximate the value function, except the parameterization of the conditional expectation should change to $\psi(\omega, k_t, \theta_t) = -\exp(P_n(\log(k_t), \log(\theta_t)))$ since utility of the agent 1 takes only negative values.
A.3 Derivation of the first order conditions for the two-sector autarky in Program 3.

The dynamic problem corresponding to the autarky with two open sectors is given by

\[
\max_{\{c_t, i_{1t}, i_{2t}\}_{t=0}^\infty} E_0 \left[ \sum_{t=0}^\infty \beta^t u(c_t) \right]
\]

subject to

\[
c_{1t} + i_{1t} + i_{2t} = f(k_{1t}) + f(k_{2t}),
\]

\[
k_{jt+1} = (1 - \delta)k_{jt} + g(i_{jt}, \theta_{t+1}), \text{ for } j = 1, 2,
\]

with \(c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0\) given.

The Lagrangian for the problem is given by

\[
\mathcal{L} = E_0 \sum_{t=0}^\infty \beta^t \{u(c_t) - \lambda [c_{1t} - \tau_t + i_{1t} + i_{2t} - f(k_{1t}) - f(k_{2t})] - \mu_{1t}(k_{1t} - (1 - \delta)k_{1t-1} - g(i_{1t-1}, \theta_t)) - \mu_{2t}(k_{2t} - (1 - \delta)k_{2t-1} - g(i_{2t-1}, \theta_t))\},
\]

where \(\lambda_{1t}, \mu_{1t}\) and \(\mu_{2t}\) are the Lagrange multipliers associated with the constraints (58) and (59). The corresponding f.o.c. are given by

\[
u'(c_t) = \lambda_t,
\]

\[-\lambda_t + \beta E_t \left[ \frac{\partial g(i_{jt}, \theta_{t+1})}{\partial i_{jt}} \right] = 0, \text{ for } j = 1, 2,
\]

\[
\lambda_t f'(k_{jt}) - \mu_{jt} + (1 - \delta)\beta E_t [\mu_{jt+1}] = 0, \text{ for } j = 1, 2.
\]

Using recursive substitution and the law of iterated expectations (62) reduces to

\[
\mu_{jt} = \beta E_t \left[ \sum_{i=0}^\infty (\beta(1 - \delta))^i f'(k_{jt+1+i})\lambda_{t+i} \right], \text{ for } j = 1, 2,
\]

which combined with (60) and (61) yields

\[
u'(c_t) = \beta E_t \left[ \frac{\partial g(i_{jt}, \theta_{t+1})}{\partial i_{jt}} \sum_{i=0}^\infty (\beta(1 - \delta))^i f'(k_{jt+1+i})u'(c_{t+i}) \right], \text{ for } j = 1, 2.
\]
Figure 1: Efficient accumulation mechanism under full enforcement.

*Note.* The simulated series correspond to the Pareto optimum equilibrium with perfect enforcement. The productivity of the foreign operated sector is set to be identical to the productivity of the domestic sector ($A = \overline{A} = 1$).
Figure 2: Comparison of the efficient accumulation mechanisms in the model with one-sector autarky.

Note. The simulated series correspond to three institutional environments: autarky equilibrium corresponding to Program 2 denoted as "au", Pareto optimum equilibrium with perfect enforcement denoted as "po", and the equilibrium with limited enforcement corresponding to Program 2 denoted as "pc1". In the environments with the foreign operated sector, its productivity is identical to the productivity of the domestic sector (A=\lambda=1).
TABLE 1. Parameterization of the models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor share of capital</td>
<td>$\alpha = 0.5$</td>
</tr>
<tr>
<td>Risk-aversion parameter of agent 1</td>
<td>$\gamma = -3$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.95$</td>
</tr>
<tr>
<td>Autocorrelation parameter of $\log(\theta_t)$</td>
<td>$\rho = 0.95$</td>
</tr>
<tr>
<td>Standard deviation of innovations of $\log(\theta_t)$</td>
<td>$\sigma_z = 0.03$</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta = 0.1$</td>
</tr>
<tr>
<td>Constants in the investment functions</td>
<td>$a = 0.6; s = 0.2; b = 0.13$</td>
</tr>
</tbody>
</table>

TABLE 2. Simulation results: the model with one-sector autarky ($\tilde{A} = 1.00$)

<table>
<thead>
<tr>
<th>Environment</th>
<th>Utility of the agent 1</th>
<th>Mean of growth rate of output (15 periods)</th>
<th>Mean of growth rate of output (35 periods)</th>
<th>Mean of capital in domestic sector</th>
<th>Increase in consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU1</td>
<td>-7.44</td>
<td>2.41%</td>
<td>1.38%</td>
<td>2.478</td>
<td></td>
</tr>
<tr>
<td>PO,PC1</td>
<td>-2.01</td>
<td>8.44%</td>
<td>3.80%</td>
<td>2.467</td>
<td>92.20%</td>
</tr>
</tbody>
</table>

Note. The environments considered are the one-sector autarky in Program 2 (AU1), the environment with perfect enforcement given in Program 1 (PO) and the environment with incentive compatibility constraint given in Program 2 (PC1). "Mean of growth rate of output" refers to the mean across independent realizations during the first 15 and 35 periods respectively. The utility of the agent 1 is measured at Time 0 and using many independent replications of the model conditioning on $\theta_0 = 1, k_0 = 1$ in case of autarky, and $k_{10} = 1, k_{20} = 0$ in case of the two sector models. The "Increase in consumption" refers to the permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other environments.
Figure 3: Comparison of the efficient accumulation mechanisms in the model with two-sector autarky; the case with no technological transfers ($A = 1.0$).

Note. The simulated series correspond to three institutional environments: autarky equilibrium corresponding to Program 3 denoted as "au", Pareto optimum equilibrium with perfect enforcement denoted as "po", and the equilibrium with limited enforcement corresponding to Program 4 denoted as "pc".
Figure 4: Comparison of the efficient accumulation mechanisms in the model with two-sector autarky; the case of technological transfers of medium magnitude ($\hat{A} = 1.10$).

Note. The simulated series correspond to three institutional environments: autarky equilibrium corresponding to Program 3 denoted as "au", Pareto optimum equilibrium with perfect enforcement denoted as "po", and the equilibrium with limited enforcement corresponding to Program 4 denoted as "pc".
Figure 5. Comparison of the efficient accumulation mechanisms in the model with two-sector autarky; the case of technological transfers of high magnitude ($\tilde{A} = 1.35$).

Note. The simulated series correspond to three institutional environments: autarky equilibrium corresponding to Program 3 denoted as "au", Pareto optimum equilibrium with perfect enforcement denoted as "po", and the equilibrium with limited enforcement corresponding to Program 4 denoted as "pc".
TABLE 3. Simulation results: the case with no technological transfers ($\tilde{A} = 1.00$)

<table>
<thead>
<tr>
<th>Model</th>
<th>Utility of the agent 1</th>
<th>Mean of growth rate of output (15 periods)</th>
<th>Mean of growth rate of output (35 periods)</th>
<th>Mean of capital in domestic/foreign sector (steady state)</th>
<th>Increase in consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU3</td>
<td>-1.861</td>
<td>2.455%</td>
<td>1.381%</td>
<td>2.470 / 2.470</td>
<td></td>
</tr>
<tr>
<td>PO</td>
<td>-1.734</td>
<td>3.035%</td>
<td>1.463%</td>
<td>2.466 / 2.466</td>
<td>3.58%</td>
</tr>
<tr>
<td>PC</td>
<td>-1.856</td>
<td>2.470%</td>
<td>1.384%</td>
<td>2.466 / 2.466</td>
<td>0.14%</td>
</tr>
</tbody>
</table>

TABLE 4. Simulation results: the case of technological transfers of medium magnitude ($\tilde{A} = 1.10$)

<table>
<thead>
<tr>
<th>Model</th>
<th>Utility of the agent 1</th>
<th>Mean of growth rate of output (15 periods)</th>
<th>Mean of growth rate of output (35 periods)</th>
<th>Mean of capital in domestic/foreign sector (steady state)</th>
<th>Increase in consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU3</td>
<td>-1.861</td>
<td>2.455%</td>
<td>1.388%</td>
<td>2.470 / 2.470</td>
<td></td>
</tr>
<tr>
<td>PO</td>
<td>-1.542</td>
<td>3.161%</td>
<td>1.523%</td>
<td>2.467 / 2.641</td>
<td>9.87%</td>
</tr>
<tr>
<td>PC</td>
<td>-1.588</td>
<td>2.787%</td>
<td>1.462%</td>
<td>2.465 / 2.639</td>
<td>8.26%</td>
</tr>
</tbody>
</table>

TABLE 5. Simulation results: the case of technological transfers of high magnitude ($\tilde{A} = 1.35$)

<table>
<thead>
<tr>
<th>Model</th>
<th>Utility of the agent 1</th>
<th>Mean of growth rate of output (15 periods)</th>
<th>Mean of growth rate of output (35 periods)</th>
<th>Mean of capital in domestic/foreign sector (steady state)</th>
<th>Increase in consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU3</td>
<td>-1.861</td>
<td>2.455%</td>
<td></td>
<td>2.470 / 2.470</td>
<td></td>
</tr>
<tr>
<td>PO</td>
<td>-1.168</td>
<td>3.450%</td>
<td></td>
<td>2.467 / 3.016</td>
<td>26.22%</td>
</tr>
<tr>
<td>PC</td>
<td>-1.168</td>
<td>3.450%</td>
<td></td>
<td>2.467 / 3.016</td>
<td>26.22%</td>
</tr>
</tbody>
</table>

Note. The models considered are the autarky in Program 3 (AU3), the model with perfect enforcement in Program 1 (PO) and the model with incentive compatibility constraint in Program 4 (PC). "Mean of growth rate of output" refers to the mean across independent realizations during the first 15 and 35 periods respectively. The utility of the agent 1 is measured at Time 0 and using many independent replications of the model conditioning on $\theta_0 = 1$ and $k_{10} = 1.1$, $k_{20} = 0.9$. The "Increase in consumption" refers to the permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other environments.