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Abstract

We embed a competitive search model with labor market discrimination, or nepotism, into a two-sector, two-country framework in order to analyze how labor market discrimination impacts the pattern of international trade and also how trade affects discrimination. Discrimination, or nepotism, reduces the matching probability and output in the skilled-labor intensive differentiated-product sector so that the country with more discriminatory firms has a comparative advantage in the simple sector. As countries alter their production mix in accordance with their comparative advantage, trade liberalization can then reinforce the negative effect of discrimination on development in the more discriminatory country and reduce its effect in the country with fewer discriminatory firms. Similarly, the profit difference between non-discriminatory and discriminatory firms increases in the less discriminatory country and shrinks in the more discriminatory one. In this way trade can further reduce discrimination in a country where it is less prevalent and increase it where it is more firmly entrenched.

JELClassification: F16, F66, J71

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1 Introduction

In Gary Becker's (1957) seminal work on labor market discrimination he suggests that discrimination is costly to the firms that practice it. In a similar way that a discriminator has to pay for his taste to exclude certain groups, a nepotist will incur a cost when he excludes non-relatives.[1] If practicing discrimination, or nepotism, reduces the relative productivity of a firm within a country, then it seems possible that a country where discrimination or nepotism is more prevalent may have lower relative productivity in sectors where exclusion is more costly. In this paper we analyze this question to determine if some forms of labor market discrimination can be a source of comparative advantage. We then return to Becker's original idea and ask whether the pro-competitive effects of international trade can mitigate discrimination.

The effect of discrimination on aggregate productivity and growth has received recent attention. For example, Hsieh et al. (2013), show that between fifteen and twenty percent of the growth in US output per worker between 1960 and 2008 can be explained by allowing blacks and white women into skilled occupations in which they were formerly very poorly represented. The negative effect of the gender wage gap on growth has also been demonstrated by Galor and Weil (1996), Lagerlöf (2003), Esteve-Volart (2009), Cuberes and Teignier (2014), and Cavalcanti and Tavares (2015).[2] The effect of nepotism on economic performance in southern European countries has been studied by Bloom and Van Reenen (2007). We depart from these previous studies by considering the effect of discrimination on the pattern of trade as well as the converse effect of trade liberalization on discrimination.

We take as given that some forms of labor market discrimination and nepotism exist and ask how does this discrimination affect the structure of the economy.[3] A very nice overview of the literature

[1] In fact, Bloom and Van Reenen (2007) show that much of the long tail of very-poorly managed firms can be explained by primogeniture.

[2] An interesting anecdotal example is provided by India. It is a democracy that does not suffer from a natural resource curse and has a large well-educated English-speaking population, but is still plagued by low labor productivity. A partial explanation may be found in the fact that twenty-five percent of India's population belongs to the scheduled (formerly backward) castes and tribes (i.e. the untouchables) and over thirteen percent of India's population is Muslim. Thus, more than thirty-eight percent of India's population has historically suffered restricted access to the Indian formal labor market.

[3] As Gary Becker noted about his (1957) book "For several years it had no visible impact on anything. Most economists did not think racial discrimination was economics, and sociologists and psychologists generally did not believe I was contributing to their fields," as quoted in Murphy (2014). The eventual realization that discrimination is an important topic for economists is echoed in the words of Kevin Murphy (2014), "Now the impact is clear. Not only

on discrimination is provided by Lang and Lehman (2012), who discuss an overwhelming number of papers that cannot reject the empirical evidence on labor-market discrimination. Fang and Moro (2010) contains a review of many additional theoretical papers on discrimination that are not covered in Lang and Lehman (2012).

To this end, we embed a directed (competitive) search model into a general equilibrium framework. There are two sectors in the economy: a simple sector that uses only labor and a sector in which each firm produces a differentiated product using labor and a manager. Firms in this second sector can only produce if they successfully hire a manager. In order to locate a manager, firms post a payment for the manager and the skilled workers decide where to apply (unskilled workers cannot become managers). Any skilled worker who does not find a match as a manager can work with the unskilled workers as labor in either sector. As we restrict entry into the differentiated product sector, firms in this sector realize profits. Part of the profits is payment to the manager, while the remainder of the profits is shared equally by all agents.

Our modeling of discrimination in a competitive search model follows Lang, Manove and Dickens (2005).[4] We start by assuming that all firms prefer to a hire a manager of a certain label. That is, productivity of either label of skilled worker is the same, but every firm has a very slight preference for an A-label over a B-label manager. Labels may refer to differences in skin color, eye color, gender, religion, caste, ancestral origin, native language, regional accent, or familial connections. This preference only matters if skilled workers of both labels apply to the same firm. In that case a firm would always hire an A-label manager and they would hire a B-label only if no A-labels apply. This firm preference implies that in equilibrium no B-labels will apply to a firm that attracts A-labels and vice-versa. Hence, there will be two posted payments in equilibrium: a higher one by firms that attract A-labels and a lower one by those that attract B-labels. Because the two groups are divided, which increases oligopsonistic power of firms in the labor market, both posted payments will be lower than in the label-blind equilibrium (i.e. in the equilibrium

is racial discrimination viewed as a subject about which economics has something useful to say, but economists are among the top academics in any field researching the topic."

[4]Although several of our results in the earlier sections are similar to theirs we present all results and propositions without referring the reader to their paper for two reasons. First, intimate familiarity with their model is necessary in order for the reader to understand our extensions, which are the inclusion of some non-discriminatory firms, a general equilibrium environment, and international trade. Second, we provide some additional figures that further explain the workings of their model and some of our additions.

without discrimination). Furthermore, because the posted payments are different, the portion of firms posting each payment will differ from the proportion of each label in the population.[5] There will then be an asymmetric arrival rate at the two groups of firms and, consequently, the overall arrival rate of potential managers at firms will be lower than in the label-blind equilibrium. Hence, the matching rate will be lower in the discriminatory equilibrium.

The lower match success rate implies fewer varieties of the aggregate differentiated product and a higher relative autarky price of this product for a country in a discriminatory equilibrium. Hence, when liberalizing trade with a label-blind country, the discriminating country will be an exporter of the simple good that does not require a manager or a skilled worker. It is through the induced distortion in the matching process that discrimination inhibits development of the differentiated product sector and generates comparative advantage in the simple sector. The country in a label-blind equilibrium will have more varieties per agent and, therefore, realized profits of a successful firm (i.e. of a firm that has found a manager) in the label-blind country will increase when they liberalize trade. Output per successful firm, output for the entire differentiated product sector, and the payment to each hired manager will also rise in the label-blind country as a result of trade liberalization. In the discriminatory country trade liberalization will have the opposite effect on profit, output, and managerial remuneration.

In order to consider the effect of trade on discrimination we introduce a second type of firm. These firms are label-blind and it is common knowledge that they do not discriminate or practice nepotism. Because they are known to not show hiring preference to either label of manager, they can offer a higher payment to B-label managers than can the existing discriminatory firms. This higher payment by a discriminatory firm would attract A-labels because they would be hired with certainty, however, they would only be hired with an equal probability by the non-discriminatory firms. The presence of these non-discriminatory firms partially mitigates the discrimination induced matching inefficiencies in the resulting equilibrium. In addition, these firms have higher expected profits than the discriminatory firms because they have a higher matching probability.[6]

[5] As a result of the lower payment and, therefore, more profit per successful match, more firms will post a payment to attract a B-label manager than those that post to attract an A-label manager.

[6] If entry were costless, then these firms would come to dominate the market which would substantiate the hypothesis first mentioned in Becker (1957) and substantiated in Arrow (1972). Alternatively, if firms had to pay an entry cost and firms had differing entry costs (or differing variable costs), then non-discriminatory firms would be able to enter for a higher entry (or variable) cost, but they would not take over the market. As our focus is on how trade affects each type

Finally, we consider trade between two countries that differ in their percentage share of non-discriminatory firms in the firm distribution. Our previous result on comparative advantage translates to this extended version of our model. In particular, the country with relatively more discriminatory firms will have a comparative advantage in the simple sector. For the country with more label-blind firms, output per successful firm, output of the differentiated product sector, the realized profit of a successful firm, and the payment to each manager will all increase when liberalizing trade and the opposite will happen in the country with more discriminatory firms. Because the expected profits of a label-blind firm are greater than those of a discriminatory firm, they will see a bigger change as a result of opening to trade. In particular, because of their higher match probability, any change in realized firm profit has a magnified effect on their expected profit. For the country with less discrimination, trade liberalization increases the label-blind firms' expected profits by more than those of the discriminatory firms. In the country with more discrimination or nepotism, trade liberalization reduces the profits of the label-blind firms by a larger amount. Hence, trade liberalization can help to reduce discrimination in the country where it is less prevalent and enhance it in the country where it is more widespread.

Our paper is related to several distinct strands of the literature.

We contribute to the research mentioned above that relates discrimination (or nepotism) to growth by considering its effect on the pattern of trade and the converse effect of trade on discrimination. Starting with Black (1995) and Rosen (1997, 2003), economists have analyzed discrimination as the equilibrium of a model with random search. Recognizing that firms may want to strategically post a payment, Lang et al. (2005) analyze discrimination as the equilibrium of a competitive search framework. We extend this literature by adding some additional non-discriminatory firms to the framework of Lang et al. (2005), embedding it into a two-sector general equilibrium environment, and allowing for international trade. Finally, our paper is related to the broad literature on international trade with labor market frictions, such as Davidson, Martin and Matusz (1999), Davidson, Matusz and Shevchenko (2008), Helpman and Itskhoki (2010), Helpman, Itskhoki and Redding (2010), Ranjan (2013), and Grossman, Helpman and Kircher (2013). We extend this literature in three ways. First, we analyze a competitive instead of a random search framework.

of firm, we limit their numbers and instead analyze how the relative profits of discriminatory and non-discriminatory firms are effected by trade.

Second, we analyze discrimination as a source of comparative advantage. Third we analyze how trade liberalization affects the prevalence of nepotism or discrimination.

In the next section we describe our basic framework. In the third section we consider the working of the model without discrimination. Discrimination is introduced in the fourth section and comparisons are made in the fifth section. International trade is considered in the sixth section. In the seventh section we introduce non-discriminatory firms and we analyze trade in this extended framework in the eighth section. Our conclusions are contained in the ninth section.

2 Economic environment

There are two countries: home and foreign. We will denote foreign variables with an $(^*)$. In each country there are two sectors. The numeraire sector produces perfectly substitutable goods with a constant returns to scale technology using only labor. The monopolistically competitive sector produces differentiated goods using labor and a manager. Upper tier preferences over goods from the two sectors can be represented by a Cobb-Douglas utility function:

$$U(C_M, C_0) = C_M^{\alpha} C_0^{1-\alpha}. \tag{1}$$

Preferences over the manufactured goods in the monopolistically competitive sector can be represented by a constant elasticity of substitution sub-utility function:

$$C_M = \left(\sum_{z=0}^{\infty} c_z^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where the elasticity of substitution between varieties is σ and $\sigma > 1$. Therefore, none of these varieties is essential to consumption. Although preferences are defined over a potentially infinite number of varieties, only a finite number will be available to consume. Agents derive income from working as either labor, or if they are skilled and successfully locate a match, as a manager. In addition, all agents are equal owners of each of the firms and they equally share any firm profits. Each firm producing in the monopolistically competitive sector has the same technology:

$$\ell_z = \begin{cases} q_z + f & if\ m_z = 1 \\ \xi q_z & if\ m_z = 0 \end{cases}, \tag{3}$$

where ℓ_z is the amount of **labor** used in producing good z, q_z is the quantity of variety z, m_z is a **manager** for the firm producing good z, f denotes the fixed input requirement, and ξ is an arbitrary large constant that makes production unfeasible if firm z is not successful in hiring a manager, i.e. if $m_z = 0$. We use the convention that all fixed costs are paid in terms of labor.

The technology for producing the numeraire good is $\ell_0 = q_0$, and the labor supply of each country, $L = L^*$, is assumed to be large enough so that there is positive numeraire production in each country and the wage of unskilled workers in either sector is, therefore, equal to the price of the numeraire good which is one.

We will be interested in the composition of firms in the monopolistically competitive sector rather than their absolute number, therefore, the number of potentially active firms in the monopolistically competitive sector, $N = N^*$, is taken as exogenous. As a result of search frictions only a fraction M of the N (M^* of the N^*) firms will be successful in hiring a manager and producing.[7] Still, the size of the economy is large enough so that the number of operating manufacturing firms is large and, therefore, the effect of each manufacturing firm's output on the price and quantity of other firms is negligible.

For each home firm that successfully hires a manager, the product market is described by monopolistic competition. As shown by Dixit and Stiglitz (1977), the set of purchased manufactured goods can be considered as a composite good C_M with corresponding aggregate price

$$P_M = \left(\sum_{z \in M} {p_z}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{4}$$

Consumer maximization of the first stage utility function yields the following demand functions:

$$C_M = \frac{\alpha I}{P_M}; \qquad C_0 = \frac{(1-\alpha)I}{P_0} \tag{5}$$

[7]Although $N = N^*$, it is not necessarily the case that $M = M^*$.

where I denotes aggregate income which we will derive below. Consumer maximization of the sub-utility function yields demand for each variety as

$$c_z = C_M \left(\frac{p_z}{P_M}\right)^{-\sigma} = \frac{\alpha I}{M p_z} \tag{6}$$

Each manufacturing firm chooses output to maximize profits, taking the output of other firms and the aggregate price index and C_M as given. This leads to the following pricing rule: $p_z = \frac{\sigma}{\sigma - 1}$. Hence,

$$c_z = \frac{\alpha I}{M \frac{\sigma}{\sigma-1}}, \tag{7}$$

and, denoting firm revenue as r_z, the gross profits of operating each firm is given by:

$$\pi_z = r_z - l_z = p_z q_z - q_z - f = p_z q_z - q_z p_z \frac{\sigma - 1}{\sigma} - f = \frac{r_z}{\sigma} - f = \frac{\alpha I}{M\sigma} - f. \tag{8}$$

Agents in each country are either skilled or unskilled. Unskilled ones work either in the numeraire sector or as laborers in the manufacturing sector. Skilled workers can work as a manager if they are offered a managerial job and they can also work as unskilled labor if their managerial search is unsuccessful.

In addition to their skill level (**skilled** versus **unskilled**), agents differ by their **label** $k \in \{A, B\}$. Labels may refer to differences in skin color, eye color, gender, religion, caste, ancestral origin, native language, regional accent, or familial connections. This label is also perfectly observable and it is common knowledge that productivity does not depend on the label. The number of skilled workers in each country with each label is given as $S_k = S_k^*$. The total number of unskilled workers in the home country is, therefore, $L - S_A - S_B = L - S$. Only a subset M of the S will find work as a manager and the remainder will work as unskilled workers in either sector.

Despite the identical productivity of all skilled workers, firms may prefer to hire an A-label manager. Formally, the preferences of firms are lexicographic: first firms prefer to hire a manager, second they prefer to hire an $A-$label manager. (An alternative, but theoretically equivalent formulation is that the disutility for a home firm that hires a B-label manager is δ, where δ is a vanishingly small amount.) Hence, if skilled managers of each label apply to the same job with the same posted bonus, then the firm will hire the A-label worker. A B-label manager will be hired by a firm only if there are no A-label skilled applicants at the posted bonus. We use the term **bonus** for the payment to managers in order to differentiate it from the payment to labor, which is the **wage**. Finally we denote the portion of B-label skilled workers in each country as β so that the number of A-label skilled workers is $S_A = (1-\beta)S$. We assume that $\beta \in (0,1)$. At times we will find it useful to write $\beta_B = \beta$ and $\beta_A = 1-\beta$ or, more generally, β_k. Firms have no preferences over the label of unskilled workers.

The timing and information structure of the model is as follows. We write the case of the home country. The foreign country is similar. First, each of the N firms posts a bonus, b_z, for a manager. Second, skilled workers observe the vector of posted bonuses, $\mathbf{b} = \{b_z\}$, and decide where to apply. Skilled workers can only apply once and to only one firm.[8] Formally, from the perspective of firms a worker's action at this stage is a collection of probabilities that they will apply to firm z, denoted as $a_z(\mathbf{b})$. The skilled worker's strategy is restricted to those that assign equal probability to all firms offering the same bonus. Hence, the workers' strategies satisfy anonymity. Third, the M firms that have an applicant are successful and will produce and sell their goods in the market. Unsuccessful firms will not produce. Unmatched skilled workers and all unskilled workers will work as wage laborers in the manufacturing or numeraire sector.

Firm z's strategy consists of posting a bonus and choosing output. Each agent's strategy is a vector of application probabilities $\mathbf{a}(\mathbf{b}) = \{a_z(\mathbf{b})\}$. If all skilled workers with the same label use the same strategy, then the expected number of workers of each label applying to firm z is given by $\lambda_{zk} = a_z(\mathbf{b})S_k$. Note that because the application probabilities sum to one we have that the market tightness S/N for each label can be expressed as $\frac{1}{N}\sum^{N}\lambda_{zk} = \beta_k S/N$. Since the firms' and the skilled workers' payoff functions depend on whether or not firms discriminate across workers, we will derive the payoff functions in the corresponding sections.

[8] As long as there is some cost to additional applications, allowing skilled workers to apply to more than one firm would not have any qualitative effect on our results.

3 Closed economy without discrimination

We start with the case of no discrimination and with a closed economy, therefore, we can suppress the subscript k in this section. We will be interested in the limiting case when N and S become very large but their ratio θ is still finite.[9] A manufacturing firm will only be able to produce if it hires a manager. This occurs if and only if it receives at least one applicant. The probability it receives at least one applicant is $1-(1-a_z)^S$. When S and N are large this converges to

$$1 - Pr\left(\lambda_z = 0\right) = 1 - (1-a_z)^S \to 1 - e^{-a_z S} = 1 - e^{-\lambda_z}. \tag{9}$$

A firm's expected profit net of payment to a manager is:

$$E\left(\pi_z^{net}\right) = \left(1 - e^{-\lambda_z}\right)\left(\pi_z - b_z\right), \tag{10}$$

where b_z denotes the bonus to the manager. The equilibrium level of b_z, which maximizes $E\left(\pi_z^{net}\right)$, will be derived below.

The probability that an applicant is hired at a firm z is the product of the probability that there is at least one applicant times the probability that they are the chosen one. Hence, the probability (from the perspective of an applicant) that they are hired at a single firm z is:

$$Pr\left(hired\right) = h(\lambda_z) = \frac{1-(1-a_z)^S}{a_z S} \to \frac{1-e^{-\lambda_z}}{\lambda_z}. \tag{11}$$

Thus, a skilled worker's expected bonus if he or she applies to a firm z is given by $V_z = b_z h\left(\lambda_z\right)$.

We now consider a sub-game perfect monopolistically competitive equilibrium (SPMCE), which is characterized as follows:

1. Each firm's b_z is a best response to the vectors of firm and skilled worker strategies, $\mathbf{b}$ and $\mathbf{a}$.

2. Each skilled worker's $a\left(\mathbf{b}\right)$ is a best response to $\mathbf{b}$ and to $\mathbf{a}\left(\mathbf{b}\right)$ of all other workers.

[9] Although we assume, for convenience, that the number of skilled workers and firms is countably infinite, we could consider them as finite if we also assume that firms know the mean of the expected number of skilled workers but do not observe the realization of S. None of our results are affected in any way by this alternative formulation.

3. Each firm chooses q_z to maximize π_z.

4. Each agent chooses C_0 and the amount c_z consumed of each variety of C_M to maximize utility subject to the budget constraint and given prices P_M and p_z.

5. Relative supply of the $M+1$ goods equals relative demand for the $M+1$ goods and the labor market clears.

Note that the large number of firms, skilled workers, and consumers ensures that $\mathbf{b}$, $\mathbf{a}$, C_M and P_M are neither sensitive to a firm's own bonus and quantity choice nor to the skilled workers' or the consumers' choices.

We will solve for the SPMCE for this game by backwards induction and we will start by showing that the skilled workers' application subgame has a unique symmetric equilibrium for each given vector of bonus offers $\mathbf{b}$. "Symmetric" refers to an equilibrium in which each skilled worker chooses the same application strategy. Since a skilled worker will only apply with positive probability at the firm(s) which offer(s) the highest bonus, the equilibrium expected bonus for a skilled worker is $V_U = \max_z \{V_z\}$, where the subscript U refers to the benchmark **unbiased** equilibrium without discrimination. Hence, in equilibrium, a firm will only receive applicants if it offers the highest bonus: $\lambda_z > 0$ and $V_z = V_U$ for $b_z \geq V_U$; $\lambda_z = 0$ and $V_z = b_z$ for $b_z < V_U$.

Thus, for $b_z \geq V_U$ we have $\lambda_z = h^{-1}\left(\frac{V_U}{b_z}\right)$. Then, for any firm choosing $b_z \geq V_U$ the expected number of applicants is λ_z. In equilibrium the expected number of applicants to all firms is:

$$\sum_{z=1}^{N} \lambda_z = \sum_{z|b_z \geq V_U} h^{-1}\left(\frac{V_U}{b_z}\right) = S. \tag{12}$$

Note that h is strictly decreasing in λ_z. Therefore, h^{-1} is strictly decreasing in V_U and the number of terms in the summand are weakly decreasing in V_U. Hence, for a given vector of bonus offers $\mathbf{b}$ there exists a unique solution V_U to the above equation. Given V_U and the vector of bonus offers $\mathbf{b}$, each λ_z follows from $\lambda_z = h^{-1}\left(\frac{V_U}{b_z}\right)$. Notice that, from the perspective of a single firm, V_U is constant and independent of the firm's own bonus offer due to the large number of firms and skilled workers.

Given this relationship between λz and b_z, we can now solve for the equilibrium of the entire wage-posting game by determining the firms' optimum bonus offers. From $V_z = b_z h(\lambda)$ we get $b_z = \frac{V_z}{h(\lambda_z)}$. Considering that $h(\lambda_z) = \frac{1-e^{-\lambda_z}}{\lambda_z}$, we can thus rewrite the expected profit net of payment to a manager as follows: $E\left(\pi_z^{net}\right) = \left(1 - e^{-\lambda_z}\right)\pi_z - \lambda_z V$. The value of λ_z which maximizes $E\left(\pi_z^{net}\right)$ results as $\lambda_z = \ln\left(\frac{\pi_z}{V(\mathbf{b})}\right)$. This latter expression can be transformed to $V(\mathbf{b}) = \frac{\pi}{e^{\lambda}}$. Considering that $V(\mathbf{b}) = bh(\lambda)$, we can derive the bonus which maximizes $E\left(\pi_z^{net}\right)$ by equating $\frac{\pi}{e^{\lambda}}$ with $bh(\lambda)$ and solving for b: $b = \frac{\pi\lambda}{e^{\lambda}-1}$. As a consequence, we can rewrite the expected equilibrium profits of a firm z, net of payments to a manager, as $E\left(\pi_z^{net}\right) = \left[1 - (1+\lambda)e^{-\lambda}\right]\pi$. Since all firms offer an identical bonus in equilibrium, potential managers apply at all firms with an identical probability, therefore, $\lambda_U = \frac{S}{N}$. Thus, we can also solve for M_U: $M_U = S\frac{1-e^{-\lambda_U}}{\lambda_U} = N(1 - e^{-\lambda_U})$.

The profit maximizing pricing rule for each single firm z is given by $p = \frac{\sigma}{\sigma-1}$. The consumers' utility maximizing consumption choices are given by the demands functions in equations (5)-(7).

In solving for market clearing, note that since all manufacturing firms charge an identical price in equilibrium, they all sell the same amount of their variety. Thus, demand for the numeraire good relative to demand for a single variety of the manufacturing good is given by: $\frac{C_0}{c} = M\frac{\sigma}{\sigma-1}\frac{1-\alpha}{\alpha}$. Labor market clearing implies that $L - Sh(\lambda) = L - M$ workers work as unskilled workers, and $M(q+f) = L_M$ of these unskilled workers work in the monopolistically competitive sector. Hence, $C_0 = L - Sh(\lambda) - L_M = L - M(1+q+f)$. The total number of skilled workers is S, therefore, the number of skilled workers who work as unskilled is $S - M = S[1 - h(\lambda)]$. The condition that relative supply equals relative demand therefore becomes: $\frac{L-M(1+q+f)}{q} = M\frac{\sigma}{\sigma-1}\frac{1-\alpha}{\alpha}$. Thus, $q_U = \alpha\frac{\sigma-1}{\sigma-\alpha}\left[\frac{L}{M_U} - (1+f)\right]$.

National income is given as the sum of the wage bill plus expected profits plus the expected payment to the managers. The $L - S$ unskilled workers each receive a wage of one. The S skilled workers have an expected return of $V + \left(1 - \frac{M}{S}\right)$, where $\frac{M}{S}$ is the probability of a successful match. The profits of the M successful firms, $\pi - b$, are shared equally by all agents and in equilibrium $V = \frac{Mb}{S}$. Hence, total income is $I = L - S + \left[V + \left(1 - \frac{M}{S}\right)\right]S + M(\pi - b) = L + M(\pi - 1)$. Substituting from equation (8) for firm profit yields $I_U = \frac{\sigma}{\sigma-\alpha}[L - (1+f)M_U]$.

Since $\pi = \frac{q}{\sigma-1} - f$, profits of an operating firm result as: $\pi = \frac{\alpha}{\sigma-\alpha}\left[\frac{L}{M_U} - (1+f)\right] - f$. We can then use this expression for π_U and $\lambda_U = \frac{S}{N}$ to solve for $E\left(\pi_U^{net}\right)$, b_U, and V_U. Then we can solve for the aggregate price index P_M and consumption of the two aggregate goods C_0 and C_M.

Finally note that $V_U = \frac{e^{-\lambda_U}}{\sigma-\alpha}\left(\alpha\left[\frac{L}{N(1-e^{-\lambda_U})}-1\right]-\sigma f\right)$ which is increasing in L and decreasing in S. Hence, if L is sufficiently large compared to S, then $V_U > 1$ and since $V_U > 1$, all skilled workers search for a managerial job.

The following proposition summarizes our results so far:

Proposition 1. *There exists a unique symmetric SPMCE in which all firms offer an identical bonus* $b_U = \frac{\pi_U \lambda_U}{e^{\lambda_U}-1}$ *and all skilled workers adopt the same mixed application strategy in which they apply at each single firm with the same probability. A single skilled worker's expected bonus is given by* $V_U = \pi_U e^{-\lambda_U}$*, profits of each operating firm result as* $\pi_U = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{N(1-e^{-\lambda_U})}-1\right]-\sigma f\right)$ *and expected profits of each firm net of bonus payments are given by* $E\left(\pi_U^{net}\right) = \left[1-(1+\lambda_U)\,e^{-\lambda_U}\right]\pi_U$*. National income results as* $I_U = \frac{\sigma}{\sigma-\alpha}\left[L-(1+f)\,M_U\right]$ *and the number of operating firms is given by* $M_U = S\frac{1-e^{-\lambda_U}}{\lambda_U} = N(1-e^{-\lambda_U})$*.*

4 Closed economy with discrimination

We now consider two labels of workers, A and B. In this section all firms prefer to hire an A-label worker if given the choice.

Firms can only post a single bonus (it is illegal to post label-dependent wages in most countries) and skilled workers can apply at most to only one firm. The skilled workers´ strategies again satisfy anonymity. A firm that attracts at least one applicant at its posted wage will successfully hire a manager. If a single firm has more than one applicant of the same label, then it will choose randomly among those applicants, however, if it has applicants from both labels, then it will always hire an A-label. As mentioned above, firms´ preferences are lexicographic. They prefer to have a match, and given a match, they prefer an A-label manager.

The case for A-label workers is similar to the previous section. Of course, the number of all skilled workers combined, S, is greater than the number of A-labels, S_A. Furthermore, the application strategies for the A-labels, a_A, will also differ from the probabilities, $a_z = 1/N$, as given in the previous section. The probability that a firm receives no A-label applicants is $Pr\,(\lambda_A = 0) = (1 - a_A)^{S_A} \to e^{-a_A S_A} = e^{-\lambda_A}$.

For B-label skilled workers, they would be hired with equal probability if and only if no A-labels apply. Hence, the probability that an additional B-applicant is hired is: $h(\lambda_A, \lambda_B) = e^{-\lambda_A} h(\lambda_B)$ $= e^{-\lambda_A} \frac{1-e^{-\lambda_B}}{\lambda_B}$.

The expected bonus for a B-label applying to a firm z is: $V_{Bz} = h(\lambda_{Az}, \lambda_{Bz})\, b_{Bz}$. The expected equilibrium bonus is $V_B = \max_z \{h(\lambda_{Az}, \lambda_{Bz})\, b_{Bz}\}$.

As in the case for A-label skilled workers, no B-label will apply to a firm which offers $b_{Bz} \leq V_B(\mathbf{b})$. Furthermore, there exists a $\bar{b}(\mathbf{b})$such that for all $b > \bar{b}(\mathbf{b})$ too many A-labels would apply and, therefore, no B-label would expect to be hired and no B-label worker would apply. Hence, $\lambda_{Bz} = 0$ for $b_z \leq V_B(\mathbf{b})$, $\lambda_{Bz} = 0$ for $b_z \geq \bar{b}(\mathbf{b})$ and $\lambda_{Bz} > 0$ only for $V_B(\mathbf{b}) < b_z < \bar{b}(\mathbf{b})$.

We now consider the firms´ optimal bonus choice. If a firm z attracts both A-label and B-label applicants, the firm's expected net profit is:

$$E\left(\pi_z^{net}\right) = E\left(\pi_A^{net}\right) + E\left(\pi_B^{net}\right) = \left(1 - e^{-\lambda_A}\right)(\pi_z - b_z) + e^{-\lambda_A}\left(1 - e^{-\lambda_B}\right)(\pi_z - b_z). \tag{13}$$

The firm's optimal choice of bonus satisfies $\frac{\partial E(\pi_z^{net})}{\partial b_z} = 0$, or

$$e^{-\lambda_A} e^{-\lambda_B} - 1 + e^{-\lambda_B} e^{-\lambda_B}\,(\pi_z - b_z)\left(\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z}\right) = 0, \tag{14}$$

however, if $\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z} < 0$, then $\frac{\partial E(\pi_z^{net})}{\partial b_z} < 0$. In this case a firm choosing a bonus that is large enough to attract A- and B-label workers would want to lower the offered bonus and then only attract B-label workers. Notice that the condition $\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z} < 0$ says that an increase in the offered bonus would decrease the number of B-label applicants by more than it would increase the number of A-label applicants. Hence, a reduction in the bonus would increase the number of B-level applicants by more than it would decrease the number of A-label applicants and no firm would ever choose a bonus that attracts both labels of potential managers.

Rewriting the term for the expected market bonus leads to: $V_B(\mathbf{b}) = b_z e^{-\lambda_A} h(\lambda_B)$ and $V_A(\mathbf{b}) =$ $b_z h(\lambda_A)$, where $h(\lambda_k) = \frac{1-e^{-\lambda_k}}{\lambda_k}$. In the appendix we show that totally differentiating these two equations with respect to the (common) bonus and holding the aggregates constant yields that $\frac{\partial \lambda_A}{\partial b_z} + \frac{\partial \lambda_B}{\partial b_z} < 0$.

We have now established the following:

Lemma 2. *In any SPMCE firms separate so that a firm chooses a bonus that will attract only A-label applicants or only B-label applicants, but not both.*

Denote by N_A and N_B the numbers of A- and B-label attracting firms and note that in equilibrium the expected net profit at each firm must be the same. Note also that $\lambda_A = \frac{S_A}{N_A}$ and $\lambda_B = \frac{S_B}{N_B}$ are the expected numbers of applicants to firms in each group. We can now derive the equilibrium bonuses, expected profits and expected income for each type of firm and label of worker. Denote π_D as the realized profit in an equilibrium with discrimination and $b_A(b_B)$ as a bonus offer that only attracts A-label (B-label) skilled workers. This leads us to proposition 3.

Proposition 3. *In any SPMCE with discrimination we have: (i)* $b_A = \frac{\pi_A \lambda_A}{e^{\lambda_A}-1}$, $V_A(\mathbf{b}) = \pi_D e^{-\lambda_A}$ *and* $E\left(\pi_A^{net}\right) = \left[1-(1+\lambda_A)e^{-\lambda_A}\right]\pi_D$*; (ii)* $\pi_D = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{M_D}-1\right]-\sigma f\right)$*, where* $M_D = S_B\frac{1-e^{-\lambda_B}}{\lambda_B} + S_A\frac{1-e^{-\lambda_A}}{\lambda_A}$*; (iii)* $b_B = V_A(\mathbf{b})$, $V_B(\mathbf{b}) = \pi_D e^{-\lambda_A}\frac{1-e^{-\lambda_B}}{\lambda_B}$ *and* $E\left(\pi_B^{net}\right) = \left(1-e^{-\lambda_B}\right)\left(1-e^{-\lambda_A}\right)\pi_D$*;*

(iv) $I_D = \frac{\sigma}{\sigma-\alpha}\left[L-(1+f)M_D\right]$.

Proof. We denote π_D as the realized profit of a firm that successfully employs a manager in the discriminatory equilibrium. Given this new notation, the derivations for parts (i) and (ii) are identical to the derivations in the case without discrimination and are shown in proposition 1. For part (iii) note that for the firms that attract B-label applicants we must have $b_B \le V_A(\mathbf{b})$ because A-label workers will apply if $b_B > V_A(\mathbf{b})$. If $b_B \le V_A(\mathbf{b})$ then only B-labels will apply. Hence, $b_B = V_A(\mathbf{b})$. Then, $V_B(\mathbf{b}) = V_A(\mathbf{b})\,h(\lambda_B) = V_A(\mathbf{b})\frac{1-e^{-\lambda_B}}{\lambda_B} = \frac{1-e^{-\lambda_B}}{\lambda_B}\pi_D e^{-\lambda_A}$ and $E\left(\pi_B^{net}\right) = \left(1-e^{-\lambda_B}\right)\left[\pi_D - V_A(\mathbf{b})\right] = \left(1-e^{-\lambda_B}\right)\left(1-e^{-\lambda_A}\right)\pi_D$.

Similar to the non-discriminatory case we can write total income as

$$I = L - S + \left[V_A + \left(1-\frac{M_A}{S_A}\right)\right]S_A + \left[V_B + \left(1-\frac{M_B}{S_B}\right)\right]S_B + M_A(\pi_D - b_A) + M_B(\pi_D - b_B) = L + M_D(\pi_D - 1).$$

Substituting from equation (8) for firm profit yields $I_D = \frac{\sigma}{\sigma-\alpha}\left[L-(1+f)M_D\right]$.

The output of a single operating firm in the monopolistically competitive sector is then given as $q_D = \alpha\frac{\sigma-1}{\sigma-\alpha}\left[\frac{L}{M_D}-(1+f)\right]$ and the profit of an operating firm is $\pi_D = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{M_D}-1\right]-\sigma f\right)$.

We can then use this expression for π_D and $\lambda_k = \frac{S_k}{N_k}$ to solve for b_k and V_k for $k \in \{a, B\}$. Then we can solve for the aggregate price index P_M and consumption of the two (aggregate) goods C_0 and C_M.

Finally note that $V_B = \frac{1-e^{-\lambda_B}}{\lambda_B} \frac{e^{-\lambda_A}}{\sigma-\alpha}\left(\alpha\left[\frac{L}{N_A(1-e^{-\lambda_A})+N_B(1-e^{-\lambda_B})}-1\right]-\sigma f\right)$ which is increasing in L and decreasing in S_A and in S_B. Hence, if L is sufficiently large compared to S, then $V_B > 1$ and, because $V_A > V_B$, we know that $V_A > 1$ so that all skilled workers will apply for a managerial position. □

We now define $\eta \equiv \frac{N_A}{N}$ and we note that $\lambda_U = \eta\lambda_A + (1-\eta)\lambda_B = \frac{S}{N}$. We can then state:

Proposition 4. *There is a unique SPMCE of this competitive search wage posting game with discrimination. In this equilibrium $E\left(\pi_A^{net}\right) = E\left(\pi_B^{net}\right)$. Furthermore, realized profit, π_D, expected profit, the realized bonuses, b_A and b_B, and the expected payoffs to the searching skilled workers, V_A and V_B are uniquely determined by λ_A and λ_B which are uniquely defined as the solution to: (i) $\lambda_B = \frac{\beta\lambda_U\lambda_A}{\lambda_A-(1-\beta)\lambda_U}$ and (ii) $\lambda_B = ln\left(\frac{1-e^{-\lambda_A}}{e^{-\lambda_A}\lambda_A}\right)$. Furthermore, both λ_A and λ_B are increasing in β and λ_U and decreasing in N.*

Proof. First note that $E\left(\pi_A^{net}\right)$ is increasing in λ_A and, therefore, is decreasing in N_A. Second note that $E\left(\pi_B^{net}\right)$ is increasing in λ_B and decreasing in λ_A and, therefore, is decreasing in N_B and increasing in N_A. Hence, in equilibrium the number of firms attracting A- and B-label applicants will adjust until $E\left(\pi_A^{net}\right) = E\left(\pi_B^{net}\right)$. Second note that using $\beta = \frac{S_B}{S_B+S_A}$ and $\lambda_U = \frac{S_A+S_B}{N}$, we can write: $\lambda_B = \frac{S_B}{N_B} = \beta\frac{\lambda_U N}{N_B} = \beta\frac{\lambda_U N}{N_B}\frac{\lambda_A}{\lambda_A} = \frac{\beta\lambda_U\lambda_A}{\frac{\lambda_A N_B}{N}}$, which we can further transform to: $\lambda_B = \frac{\beta\lambda_U\lambda_A}{\frac{\lambda_A(N-N_A)}{N}} = \frac{\beta\lambda_U\lambda_A}{\lambda_A-\frac{\lambda_A N_A}{N}} = \frac{\beta\lambda_U\lambda_A}{\lambda_A-\frac{S_A}{N}} = \frac{\beta\lambda_U\lambda_A}{\lambda_A-(1-\beta)\lambda_U}$. Third, note that from $E\left(\pi_A^{net}\right) = E\left(\pi_B^{net}\right)$ it follows that $1-(1+\lambda_A)e^{-\lambda_A} = \left(1-e^{-\lambda_B}\right)\left(1-e^{-\lambda_A}\right)$, which we can transform to $e^{-\lambda_B}\left(1-e^{-\lambda_A}\right) = \lambda_A e^{-\lambda_A}$ and further to $\lambda_B = \ln\left(\frac{1-e^{-\lambda_A}}{\lambda_A e^{-\lambda_A}}\right)$.

From equation (i) we have $\frac{\partial\lambda_B}{\partial\lambda_A} = -\frac{\beta(1-\beta)\lambda_U^2}{[\lambda_A-(1-\beta)\lambda_U]^2}$, which is negative and defined as long as $\lambda_A \neq (1-\beta)\lambda_U$. In addition, $\frac{\partial^2\lambda_B}{(\partial\lambda_A)^2} = \frac{2\beta\lambda_U^2(1-\beta)}{[\lambda_A-(1-\beta)\lambda_U]^3}$. Hence, $\frac{\partial^2\lambda_B}{(\partial\lambda_A)^2} > 0$ if $\lambda_A > (1-\beta)\lambda_U$ and $\frac{\partial^2\lambda_B}{(\partial\lambda_A)^2} < 0$ if $\lambda_A < (1-\beta)\lambda_U$. Second, considering equation (ii), we can derive the following: $\frac{\partial\lambda_B}{\partial\lambda_A} = \frac{\lambda_A-1+e^{-\lambda_A}}{(1-e^{-\lambda_A})\lambda_A} > 0$. Equation (ii) is positive for all values of $\lambda_A \geq 0$ and equation (i) is positive for $\lambda_A > (1-\beta)\lambda_U$. Note that $\lambda_A > (1-\beta)\lambda_U$ is equivalent to $N_A < N$ which must hold since $\beta \in (0,1)$. Hence, there is a unique solution for λ_A, λ_B where both are greater than zero. This solution is illustrated in figure 1.

Note that if $N = N_A + N_B$ increases, then the curve illustrating $\lambda_B = \frac{\beta\lambda_U\lambda_A}{\lambda_A-(1-\beta)\lambda_U}$ in figure 1 shifts downwards (which follows from: $\frac{\partial\lambda_B}{\partial N} = \frac{\partial\lambda_U}{\partial N}\frac{\beta\lambda_A^2}{[\lambda_A-(1-\beta)\lambda_U]^2} < 0$). The result is shown in figure 2. Thus, if N increases, the equilibrium levels of both λ_A and λ_B decrease. Similarly, from equation (i) we have $\frac{\partial\lambda_B}{\partial\beta} > 0$ and that $\frac{\partial\lambda_B}{\partial\lambda_U} > 0$, so that equation (i) shifts up and both λ_A, λ_B are increasing in β and in λ_U. □

The essence of the proof of proposition 4 and the determination of λ_A and λ_B can be seen with the help of the following figure 1. The relationship between λ_A and λ_B and β and λ_U are shown in figure 2.

Figure 1: Determination of λ_A and λ_B

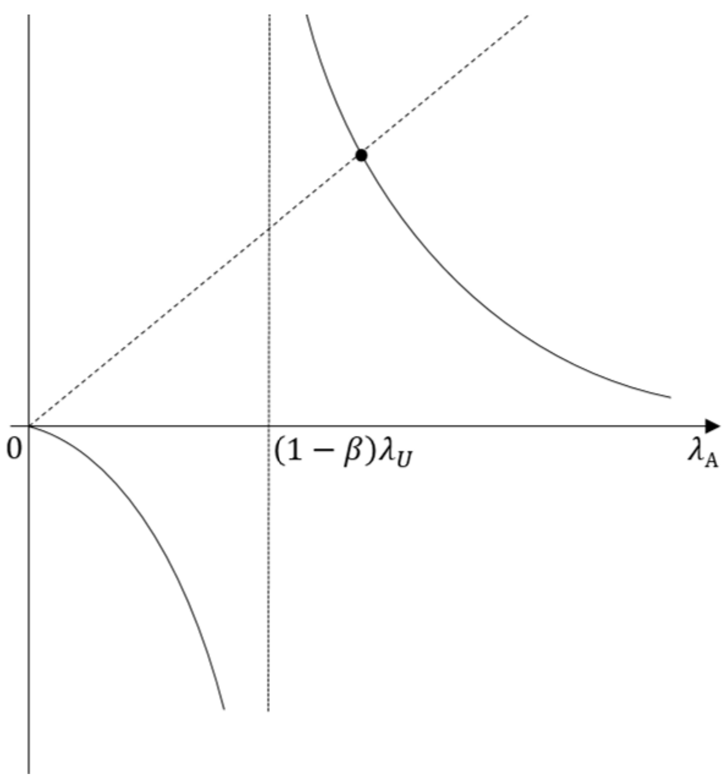
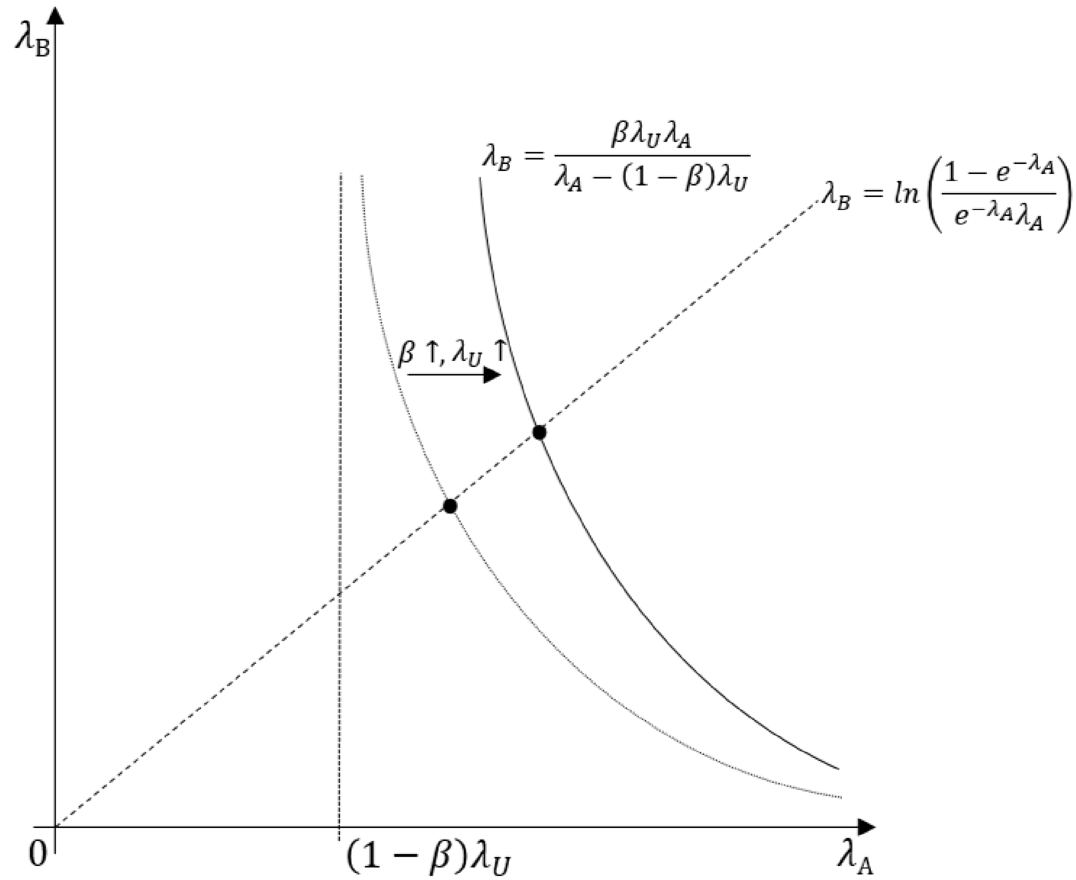


Figure 2: Comparative statics of the autarkic equilibrium with discrimination



From the results of proposition 4 we can derive the following useful lemma.

Lemma 5. *In the unique SPMCE of the competitive search discriminatory wage posting game* $\lambda_B < \lambda_U < \lambda_A$.

Proof. This result follows from $\lambda_B = \ln\left(\frac{1-e^{-\lambda_A}}{e^{-\lambda_A}\lambda_A}\right)$, which can be transformed to $\lambda_B - \lambda_A = \ln\left(\frac{1-e^{-\lambda_A}}{\lambda_A}\right) = \ln\left(h\left(\lambda_A\right)\right) < 0$. Thus, $\lambda_B = \frac{S_B}{N_B} < \frac{S_B+S_A}{N_B+N_A} = \lambda_U < \frac{S_A}{N_A} = \lambda_A$. □

This lemma is important because it allows us to compare the discriminatory and non-discriminatory equilibrium.

5 Comparing equilibria

When comparing the discriminatory to the non-discriminatory equilibrium, the most important variables are the arrival rate of applicants at the firms and the number of successful matches. Using the result on the arrival rates from lemma 5 we can analyze the number of successful matches by considering the inverse problem of the number of vacancies. We define the average vacancy rate in the discriminatory equilibrium as:

$$\Psi(\eta) = \frac{N_A}{N} e^{-\lambda_A} + \frac{N_B}{N} e^{-\lambda_B} = \eta e^{-\frac{S_A}{\eta N}} + (1-\eta)\, e^{-\frac{S_B}{(1-\eta)N}}. \tag{15}$$

We now show that $\Psi(\eta)$ is strictly convex in η, that $\Psi(\eta)$ attains its minimum at $\eta_{min} = \frac{S_A}{S_A+S_B}$ and that $\Psi(\eta_{min}) = e^{-\frac{S_A+S_B}{N}} = e^{-\lambda_U}$. The partial derivative of Ψ with respect to η results as: $\frac{\partial \Psi}{\partial \eta} = e^{-\frac{S_A}{\eta N}} \left(1 + \frac{S_A}{\eta N}\right) - e^{-\frac{S_B}{(1-\eta)N}} \left(1 + \frac{S_B}{(1-\eta)N}\right)$. Note that $\frac{\partial \Psi}{\partial \eta} = 0$ if $\eta = \frac{S_A}{S_A+S_B}$. To see that $\eta = \frac{S_A}{S_A+S_B}$ is, in fact, a minimizer of Ψ, note that $\frac{\partial^2 \Psi}{\partial \eta^2} = e^{-\frac{SA}{\eta N_D}} \lambda_A^2 \frac{N_D}{N_A} + e^{-\frac{SB}{(1-\eta)N_D}} \lambda_B^2 \frac{N_D}{N_B} > 0$. Substitution then yields that $\Psi(\eta_{min}) = e^{-\frac{S_A+S_B}{N}}$, which equals the vacancy rate in the non-discriminatory case since $\frac{S_A+S_B}{N} = \lambda_U$. We state this result as proposition 6 below.

Proposition 6. *The number of vacancies is larger, and the number of successful matches is smaller, in the discriminatory equilibrium.*

Proposition 6 is an important result because it will allow us to show that there is less production of the monopolistically competitive good and more of the numeraire good in a discriminatory equilibrium. We then use this result to derive the pattern of comparative advantage. Before we consider international trade we compare expected profits, bonuses, and expected income in the discriminatory and non-discriminatory equilibrium.

Using propositions 1 and 3, and denoting with a subscript e the type of equilibrium we are considering we can rewrite the realized profit of a successful firm as $\pi_e = \frac{\alpha}{\sigma-\alpha}\left(\frac{L}{M_e} - 1\right) - \frac{\sigma f}{\sigma-\alpha}$. After substituting income into the demand for a variety (from equation 7) we can write the equilibrium output of a successful firm in either type of equilibrium as $q_e = \frac{(\sigma-1)\alpha}{\sigma-\alpha}\left[\frac{L}{M_e} - (1+f)\right]$.

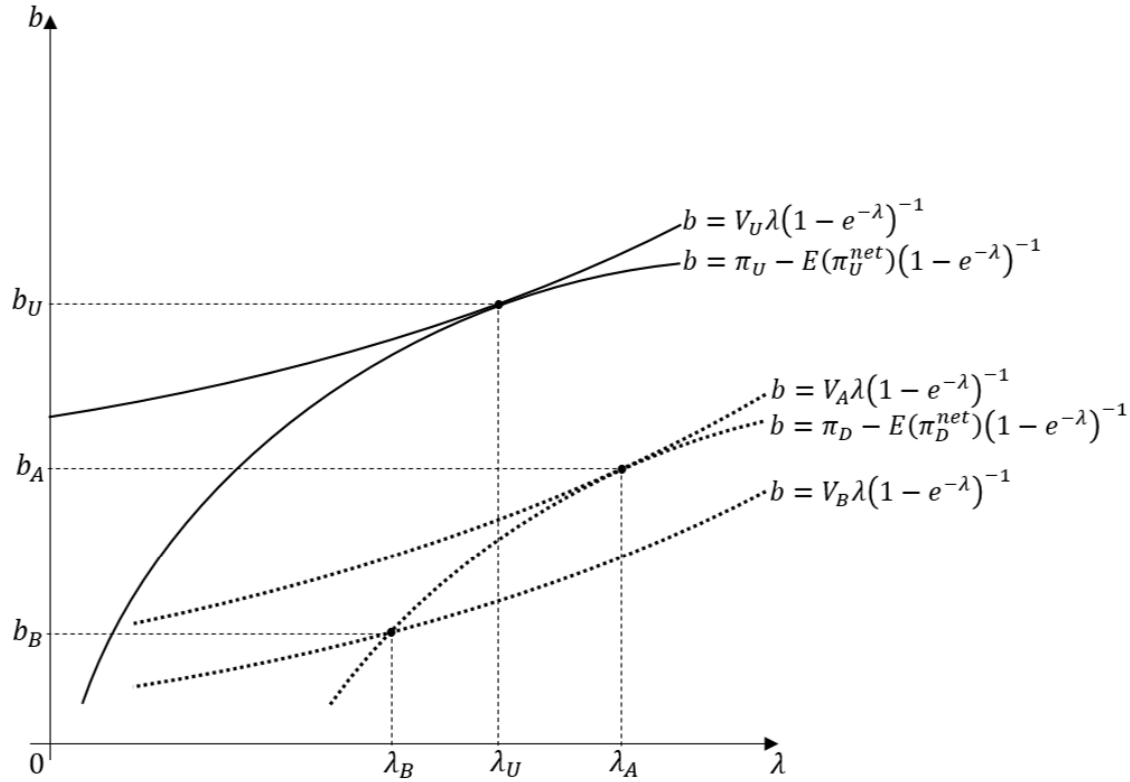
Now from proposition 6 we know that $M_D < M_U$, therefore, the realized profit and output of a successful firm is higher in the discriminatory equilibrium: $\pi_D > \pi_U$ and $q_D > q_U$. This result is intuitive. If there are less successful firms, then there is less competition and the profits of each producing firm is greater. In comparing expected profits in the discriminatory and non-discriminatory equilibrium note that $E\left(\pi_A^{net}\right) = E\left(\pi_B^{net}\right)$ in equilibrium. Hence, we only need to compare $E\left(\pi_A^{net}\right) = \left[1 - (1+\lambda_A)\, e^{-\lambda_A}\right] \pi_D$ in the discriminatory case to $E\left(\pi_U^{net}\right) = \left[1 - (1+\lambda_U)\, e^{-\lambda_U}\right] \pi_U$ from the non-discriminatory case. Now, $1 - (1+\lambda)\, e^{-\lambda}$ is increasing in λ and from lemma 5 we know that $\lambda_A > \lambda_U$. Hence, given that $\pi_D > \pi_U$ we know that the expected profits are also larger in a discriminatory equilibrium. We summarize these results in proposition 7.

Proposition 7. *Expected and realized firm profits, and output of each variety, are larger in the discriminatory equilibrium.*

The overall effect on workers is not as easy to disentangle. The change in λ produces two opposing effects on skilled workers. First, with respect to A-label workers, note that holding π constant, b_A and $V_A(\mathbf{b})$ are both decreasing in λ. Hence, given that $\lambda_A > \lambda_U$, if π does not change, then the bonuses and expected incomes of A-labeled skilled workers would be lower in the discriminatory equilibrium. Of course, as shown in proposition 7 the profit of each successful firm would be higher in the discriminatory equilibrium and part of this profit would be passed on to the manager in their bonus. With respect to the B-label managers note that they have a lower bonus and expected income than A-labels. Their bonus is lower because $b_B = V_A(\mathbf{b}) = h(\lambda_A)\, b_A$ and $h(\lambda_A) < 1$. In addition, their expected income is lower since $V_B(\mathbf{b}) = h(\lambda_B)\, b_B = h(\lambda_B)\, V_A(\mathbf{b}) < V_A(\mathbf{b})$.

In figure 3 we see a depiction of the discriminatory and non-discriminatory equilibria (for the case when the realized firm profit does not rise enough to increase the expected bonus of the skilled workers). The topmost tangency between the firm's iso-profit and the skilled workers indifference curve indicates the non-discriminatory equilibrium at (λ_U, b_U). In the discriminatory equilibrium the firm has higher profits and this is reflected by movement to an iso-profit that lies to the southeast of the non-discriminatory equilibrium iso-profit. In the resulting discriminatory equilibrium the A-labels are on a lower indifference curve, with a lower bonus and a lower probability of finding a match (a larger λ). The B-labels are on an even lower indifference curve with a much lower bonus but a greater probability of successfully finding a match. The firm's profit is the same whether or not they post a bonus to attract A- or B-label managers. We will return to this figure in a later section when we introduce some non-discriminatory firms.

Figure 3: The equilibrium with and without discrimination



6 Trade liberalization

We now consider international trade. In order to analyze the pattern of trade we begin by deriving the autarky prices for the home economy, which is in a discriminatory equilibrium, and the foreign economy which is initially assumed to be in a label-blind equilibrium. Given that $L = L^*$ and $S = S^*$, we know from proposition 6 that the number of matches is lower in the home country. In particular, $M_D < M_U = M^*$. Hence, production of the monopolistically competitive good is lower in the home country. Given that the vacancy rate is higher, and $L = L^*$, the production of the numeraire good must be larger in the home country. From equations (4) through (7) we can then write the relative autarky prices in the home and foreign countries as:

$$\frac{P_M}{P_0} = \frac{\alpha}{1-\alpha}\frac{C_0}{C_M} = M_D^{\frac{1}{1-\sigma}} p_z > (M^*)^{\frac{1}{1-\sigma}} p_z = \left(\frac{P_M}{P_0}\right)^* . \tag{16}$$

We have now established the following result.

Proposition 8. *The country in the discriminatory equilibrium has a comparative disadvantage in the manufacturing sector.*

We now consider how trade liberalization affects the home and the foreign country. As its manufacturing output falls below the foreign country's manufacturing output, the relative size of the home country's numeraire sector can grow. In addition, the total number of available varieties increases. This produces two counteracting effects: first, the price index P_M, and a firm's profits, are both decreasing in the number of available varieties; second, a firm's profits are increasing in the market size. In the case where the countries are symmetric, these two effects cancel out: $\pi_e^{autarky} = \frac{\alpha}{\sigma-\alpha}\left(\frac{L}{M_e} - 1\right) - \frac{\sigma f}{\sigma-\alpha} = \frac{\alpha}{\sigma-\alpha}\left(\frac{2L}{2M_e} - 1\right) - \frac{\sigma f}{\sigma-\alpha} = \pi_e^{trade}$.

In the case in which countries are asymmetric because the home country is in a discriminatory equilibrium we have that for home country firms:

$$\pi_D^{trade} = \frac{\alpha}{\sigma-\alpha}\left(\frac{2L}{M_D + M^*} - 1\right) - \frac{\sigma f}{\sigma-\alpha} < \frac{\alpha}{\sigma-\alpha}\left(\frac{L}{M_D} - 1\right) - \frac{\sigma f}{\sigma-\alpha} = \pi_D^{autarky} \tag{17}$$

because $M_D < M_U = M^*$. Finally we consider output of each firm. Comparing output in trade and autarky we have:

$$q_D^{trade} = \frac{(\sigma-1)\,\alpha}{\sigma-\alpha}\left(\frac{2L}{M_D + M^*} - (1+f)\right) < \frac{(\sigma-1)\,\alpha}{\sigma-\alpha}\left(\frac{L}{M_D} - (1+f)\right) = q_D^{autarky}. \tag{18}$$

The output of the monopolistically competitive sector is $M_D q_D$ and is, therefore, also lower in trade than in autarky. We have now established the following proposition.

Proposition 9. *When liberalizing trade, the output of each manufacturing firm, of the manufacturing sector, and the realized and expected firm profits all fall in the discriminatory country and the output of the numeraire sector increases. The opposite results occur in the label-blind country.*

Finally, we use our results for firm profits to note the effect of trade liberalization on skilled workers of both labels. From proposition 3 we see that the change in the skilled workers bonuses and expected incomes with respect to profits can be written as

$$\frac{\partial b_B}{\partial \pi} = h(\lambda_A)\frac{\partial b_A}{\partial \pi}; \quad \frac{\partial V_B}{\partial \pi} = h(\lambda_B)\frac{\partial V_A}{\partial \pi}. \tag{19}$$

It is straightforward to see that bonuses and expected incomes of both labels of skilled workers are increasing in realized firm profits and, because $h(\lambda) < 1$, that they are increasing faster for A-labels. Hence, the decrease in manufacturing firm profits that is driven by the change from autarky to free trade is felt more strongly by the A-label than by the B-label skilled workers. We summarize this discussion in proposition 10.

Proposition 10. *When liberalizing trade the equilibrium bonuses and expected income of skilled workers decrease in the discriminatory country and increase in the label-blind country. The change is larger for A-label than for B-label skilled workers.*

An interesting implication of proposition 10 is that trade liberalization reduces the remuneration gap (among properly matched skilled workers) in the discriminatory country but increases this gap in the label-blind country.

7 Co-existence of discriminating and non-discriminating firms in autarky

We now consider the case in which some non-discriminating firms $N_0 < \beta N$ are also in the home country. For these firms the label is irrelevant. Hence, when faced with both an A-label and a B-label managerial applicant each applicant is hired with equal probability. In order to continue to have a clear conception of comparative advantage we assume that the total number of home firms is still N so that $N_D = N - N_0$ is the number of discriminatory or nepotistic firms. [10]

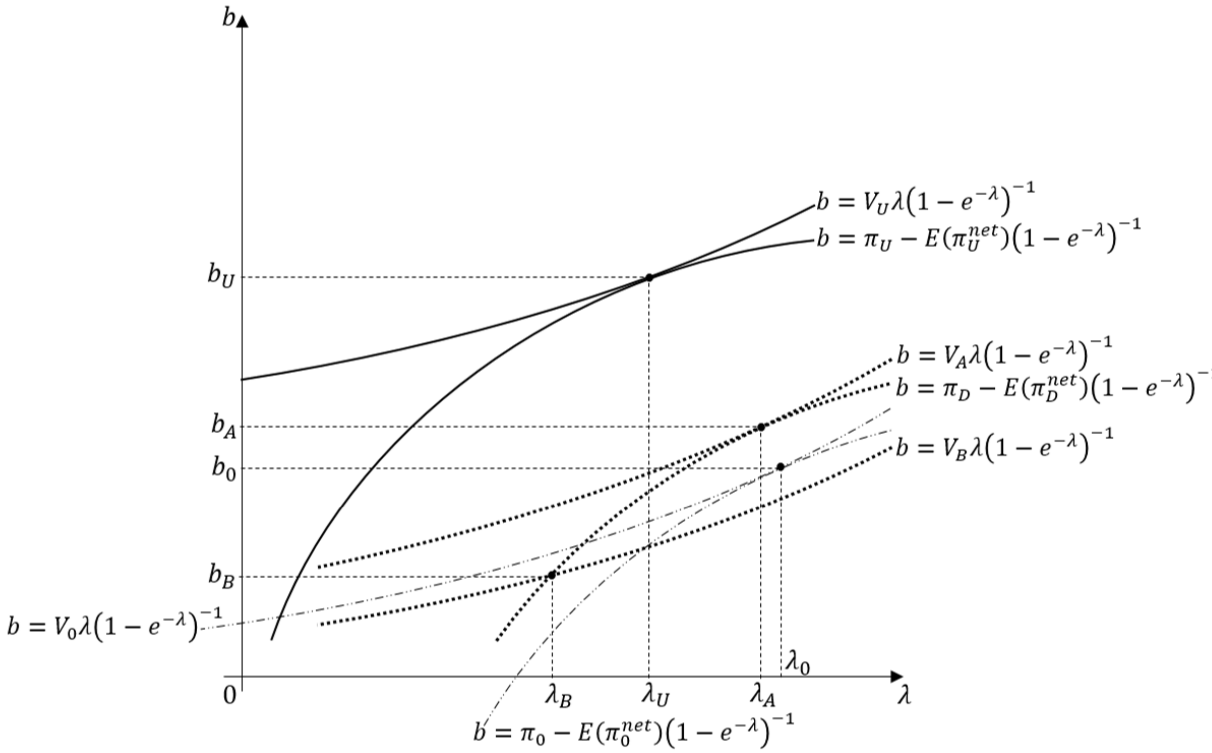
[10] If we allowed for free entry, then we could no longer be certain of $N = N^*$ and the total number of matches, as well as the pattern of comparative advantage, would no longer be a simple mapping from the vacancy rate. In particular, it would also depend on the shape of the distribution driving the firm heterogeneity. Although (as will be seen below), a

To develop the intuition for the results that are introduced in this section we refer the reader again to figure 3. In figure 3 we illustrate the equilibrium without non-discriminatory firms. We see there that the low bonus offered to the B-label applicants generates "too many" firms posting that low bonus in the attempt to attract a B-label manager.[11] Hence, the inefficiency illustrated in figure 3 suggests that a firm that is known not to discriminate could post a bonus (and a corresponding hiring probability) that would attract B and not A-label applicants. A discriminating firm could not post such a bonus (and expect only B-label applicants) because it is known that they would show priority to A-label applicants. This bonus is shown in figure 4.

label-blind firm would have larger expected profits than a discriminating one with an equivalent cost, as long as there is some firm heterogeneity (in either fixed or variable costs), then the two types of firms would coexist with free entry. In particular, the cutoff productivity level (fixed cost) would be lower (higher) so that the average cost of the marginal label-blind firm would be higher than that of the marginal discriminatory firm. Even though the pattern of comparative advantage does not admit a simple solution in such an environment we could make predictions on how the number of each type of firm responds to trade liberalization. We save this extension for future research.

[11] $\lambda_B < \lambda_A$ implies $\frac{\beta S}{N_B} < \frac{(1-\beta)S}{N_A}$ or $\frac{\beta}{1-\beta} < \frac{N_B}{N_A}$, therefore, we say "too many" firms post for B-labels.

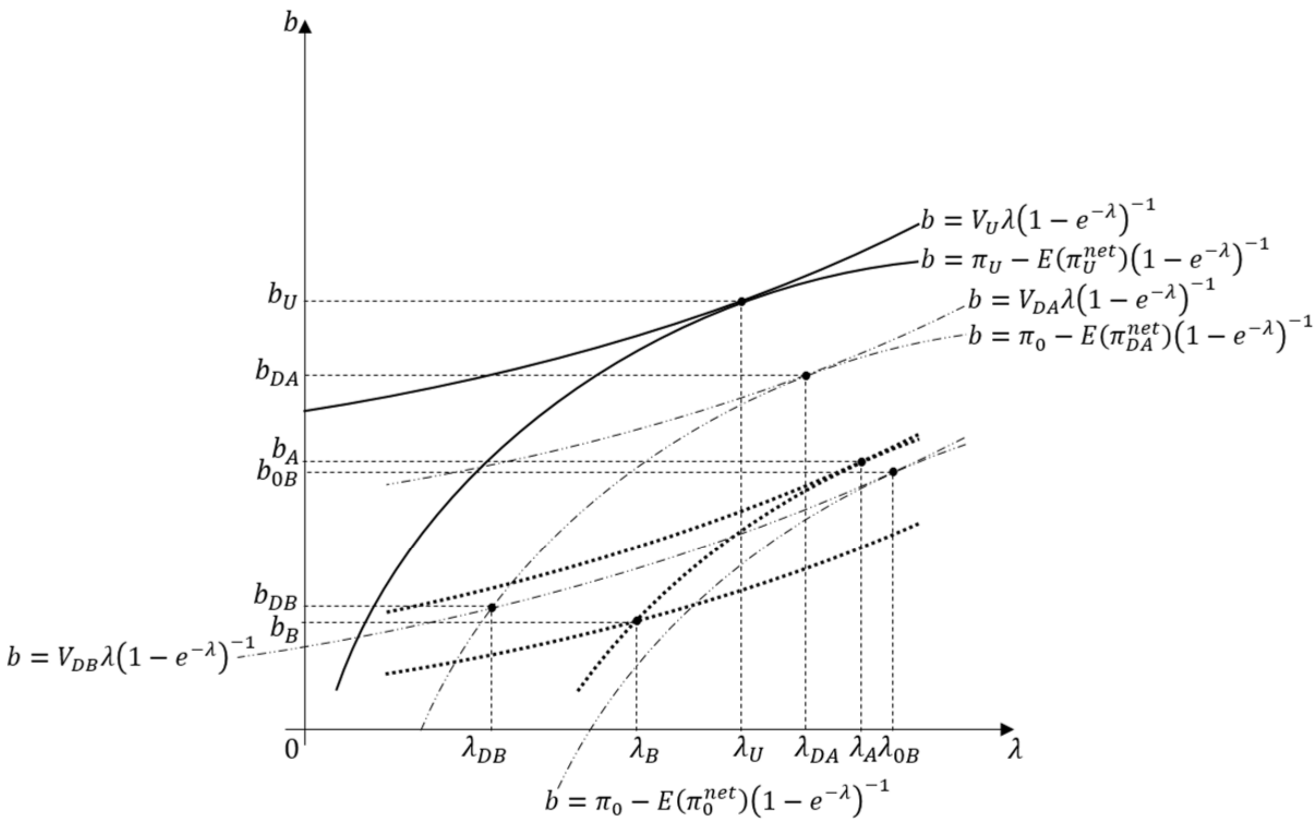
Figure 4: The potential for non-discriminatory firms



In figure 4, at b_0 the B-label applicant is on a higher indifference curve and the non-discriminatory firms have larger profits than the discriminatory ones. Of course, figure 4 does not depict the new equilibrium. For a B-label to be indifferent between a label-blind and a discriminatory firm the bonus offered by the discriminating firms to the B-label workers must increase as well. In response to the higher bonus required to attract the B-label applicants (and the resulting fewer applicants at the discriminatory B-label firms) some discriminatory firms switch from attracting B-label applicants to attracting A-label applicants.

The new coexistence equilibrium is depicted by the dashed lines in figure 5 (along with the unbiased equilibrium one with solid lines and the discriminatory equilibrium with dotted lines). We see there that the expected profits for discriminatory firms decrease and the expected payment of both labels of applicants increases. Not only does the bonus offered to both labels of applicants increase, but the applicant to position ratio decreases for both types of discriminatory firms.

Figure 5: The autarky equilibrium with co-existence of discriminating and non-discriminating firms



Before proceeding with the formal analysis we introduce the following notation. Of the N_D firms N_{DA} will attract only A-labels and N_{DB} will attract only B-labels in the coexistence equilibrium. Similarly, N_{0A} and N_{0B} are the number of label-blind firms attracting only A and only B-labels in the coexistence equilibrium. Of the skilled workers, S_{DA} and S_{DB} are the numbers that apply to the discriminatory firms and S_{0A} and S_{0B} are the numbers that apply to the non-discriminatory firms. The extension to λ_{DA}, λ_{DB}, λ_{0A}, and to λ_{0B} is straightforward. More generally we can write N_{tk}, S_{tk}, and λ_{tk} where $t \in \{D, 0\}$ and k signifies the label of worker attracted by that type t firm. Similarly, b_{tk} is the bonus offered by a type t firm attracting a label-k manager and V_{tk} is the expected payment.

It will prove useful to consider the skilled workers that apply to the discriminatory firms. The percentage of B-labels that apply to the discriminatory firms is $\beta_D = \frac{S_{DB}}{S_{DA}+S_{DB}}$ and the average arrival rate of applicants at discriminatory firms is $\lambda_D = \frac{S_{DA}+S_{DB}}{N_{DA}+N_{DB}}$.

We write M_0, π_0, and I_0 for the number of matches, the realized firm profit, and aggregate income in the equilibrium with non-discriminatory firms. Finally, we write $E\left(\pi_{00}^{net}\right)$ for the expected profits of the non-discriminatory firms and $E\left(\pi_{DA}^{net}\right)$ and $E\left(\pi_{DB}^{net}\right)$ for that of the discriminatory firms. Of course, in equilibrium the expected profits of all the non-discriminatory firms are equal and we write $E\left(\pi_{0D}^{net}\right) = E\left(\pi_{DA}^{net}\right) = E\left(\pi_{DB}^{net}\right)$.

Note that the restriction on N_0 indicates that if the label-blind firms post to attract only B-labels and the discriminatory ones post for all the A-labels then we would have $\lambda_{0B} > \lambda_U > \lambda_{DA}$ so it could not replicate the unbiased equilibrium. We now establish the composition of the firms in any equilibrium with $N_0 < \beta N$ non-discriminatory firms coexisting with $N_D = N - N_0$ discriminatory ones.

Lemma 11. *In any equilibrium where $N_0 < \beta N$ non-discriminatory firms coexist with $N_D = N - N_0$ discriminatory firms all of the label-blind firms post the same bonus, $b_{0B} > b_{DB}$, and attract only B-label applicants. Hence, $N_{0B} = N_0$, $N_{0A} = 0$ and, therefore, $S_{DA} = S_A$ and $N_{DA} > 0$.*

Proof. We start by showing that N_{DA}, N_{DB}, N_{0A}, and N_{0B} cannot all simultaneously be positive. First note that $N_{tk} > 0$ in equilibrium if and only if they attract some skilled workers so that $S_{tk} > 0$, therefore, λ_{tk} would be strictly positive and finite. Second, note that if $N_{tk} > 0$ then any posted bonus by the non-discriminating firms must leave label k applicants indifferent between the non-discriminating and the discriminating firms in equilibrium. For the A-label applicant this indifference implies $V_{0A} = V_{DA} = \pi_0 e^{-\lambda_{DA}}$. For the B-label applicants this indifference implies $V_{0B} = V_{DB} = \pi_0 \frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}} e^{-\lambda_{DA}}$. Third, note that if both $N_{0B} > 0$ and $N_{0A} > 0$, then applicants must be indifferent between either group of label-blind firms. Putting $V_{0A} = V_{0B}$ and using the above relationships implies that $\pi_0 e^{-\lambda_{DA}} = \pi_0 \frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}} e^{-\lambda_{DA}}$ or $1 = \frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}$, which is impossible given that $1 - e^{-\lambda_{DB}} < \lambda_{DB}$ for any $\lambda_{DB} \in (0, \infty)$.

We now show that $N_{0B} > 0 = N_{0A}$. To see this point, note that the discriminatory firms that attract B-label applicants maximize expected profit subject to the constraint that the bonus for B-labels is no larger than the expected payoff of the A-labels. This constraint arises because an A-label applicant would always be hired instead of a B-label at any discriminatory firm. A label-blind firm does not face this constraint and because the derivative of expected profits with respect to the bonus is positive at $b_{0B} = V_{DA}(\mathbf{b}) < b_{DA}$ the non-discriminatory firms can increase profits by offering a higher bonus, $b_{0B} > b_{DB}$, to B-label applicants. On the other hand, when attracting A-label applicants, the bonus b_{DA} is profit maximizing. Hence, given that all discriminatory firms earn the same profit, that the non-discriminatory firms cannot earn higher profits than the discriminatory firms if they attract A-labels, that they can earn higher profits if they attract B-labels, and that $N_0 < \beta N < N_B$ (because $\lambda_B = \frac{\beta S}{N_B} < \frac{S}{N} = \lambda_U$) we must have $N_{0B} = N_0 > 0 = N_{0A}$. Furthermore, because these $N_0 = N_{0B}$ firms are identical and cannot coordinate their actions we have that they all choose the same $b_{0B} > b_{DB}$. Finally, given that $N_{0A} = 0$ and that $S_A > 0$ it must be the case that $N_{DA} > 0$. □

We now show that there is a unique equilibrium with the coexistence of discriminatory and non-discriminatory firms. In this equilibrium some of the discriminatory firms continue to attract B-labels, so that $N_{DB} > 0$. In addition, the profit of the label-blind firms is strictly larger than that of the discriminatory firms.

Proposition 12. *There exists a unique equilibrium with $N_0 < \beta N$ non-discriminating firms and $N_D = N - N_0$ discriminating firms. In this equilibrium $0 = \lambda_{0A}$, $0 < \lambda_{DB} < \lambda_B$, $0 < \lambda_{DA} < \lambda_A$, $\lambda_{DB} < \lambda_{0B}$, and $E\left(\pi_{00}^{net}\right) > E\left(\pi_{0D}^{net}\right)$.*

Proof. The equilibrium is defined as follows. The expected profits of the discriminatory firms must be equal so that

$$E\left(\pi_{DA}^{net}\right) = \left[1 - (1 + \lambda_{DA}) e^{-\lambda_{DA}}\right] \pi_0 = \left(1 - e^{-\lambda_{DB}}\right)\left(1 - e^{-\lambda_{DA}}\right) \pi_0 = E\left(\pi_{DB}^{net}\right). \tag{20}$$

If $N_{DB} = 0$, then equation (20) would become $\left[1-(1+\lambda_{DA})\, e^{-\lambda_{DA}}\right]\pi_0 < \left(1-e^{-\lambda_{DA}}\right)\pi_0$ so that a single discriminatory firm could increase their expected profit by the choice of a hiring probability (and bonus) that would attract a B-label.

Note that equation (20) can be transformed to

$$\lambda_{DB} = \ln\left(\frac{1-e^{-\lambda_{DA}}}{\lambda_{DA}e^{-\lambda_{DA}}}\right) \tag{21}$$

which is comparable to equation (ii) in proposition 4. Next, using the expressions $\beta_D = \frac{S_{DB}}{S_{DA}+S_{DB}}$ and $\lambda_D = \frac{S_{DA}+S_{DB}}{N_{DA}+N_{DB}}$ an equation similar to equation (i) in proposition 4 can be derived as:

$$\lambda_{DB} = \frac{\beta_D\lambda_D\lambda_{DA}}{\lambda_{DA}-(1-\beta_D)\,\lambda_D}. \tag{22}$$

In addition, the B-label agents must be indifferent between applying to a non-discriminatory and a discriminatory firm:

$$V_{0B} = b_{0B}\frac{1-e^{-\lambda_{0B}}}{\lambda_{0B}} = \frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}e^{-\lambda_{DA}}\pi_0 = V_{DB}. \tag{23}$$

From lemma 11 we know that $b_{0B} > e^{-\lambda_{DA}}\pi_0 = b_{DB}$ and because $h(\lambda)$ is declining in λ it is, therefore, seen that $\lambda_{0B} > \lambda_{DB}$. From lemma 11 we also know that the label-blind firms do not face the same constraint as the discriminatory firms and by increasing $b_{0B} > b_{DB} = V_{DA}$ they have larger expected profits than the discriminatory ones so that $E\left(\pi_{00}^{net}\right) > E\left(\pi_{0D}^{net}\right)$.

For a given λ_{0B} (which is a function of the yet to be determined S_{0B} or equivalently β_D) the profit maximizing bonus of the non-discriminatory firms can be derived in a manner similar to that in propositions 1 and 3 as:

$$b_{0B} = \frac{\lambda_{0B}\pi_0}{e^{\lambda_{0B}}-1} \tag{24}$$

Equations (23) and (24) can be solved for λ_{0B} as a function of λ_{DB} and λ_{DA}:

$$\lambda_{0B} = \ln\left(\frac{\lambda_{DB}e^{\lambda_{DA}}}{1-e^{-\lambda_{DB}}}\right) \tag{25}$$

Equations (21), (22), and (25) jointly determine the three variables λ_{0B}, λ_{DA} and λ_{DB}. As in proposition 4, equation (21) describes a positively sloped curve in a diagram like in figure 1, but with λ_{DB} and λ_{DA} on the axes. Equation (22) describes a negatively sloped and strictly convex curve in a diagram like in figure 1 as long as $\lambda_{DA} > (1-\beta_D)\lambda_D$, which is equivalent to $\frac{S_A}{N_{DA}} > \frac{S_A}{N_{DA}+N_{DB}}$, which is true. Thus, λ_{DB} and λ_{DA} are uniquely defined for a given level of S_{DB}.

To determine how λ_{DB} and λ_{DA} depend on S_{DB}, we consider equations (21) and (22). First, note that equation (21) does not depend on S_{DB}. Second, consider the following partial derivative from equation (22):

$$\frac{\partial \lambda_{DB}}{\partial S_{DB}} = \frac{\frac{\partial \beta_D}{\partial S_{DB}}\left(\lambda_{DA}-\lambda_D\right)\lambda_D + \frac{\partial \lambda_D}{\partial S_{DB}}\beta_D\lambda_{DA}}{\left[\lambda_{DA}-\left(1-\beta_D\right)\lambda_D\right]^2}\lambda_{DA},$$

which is positive since $\lambda_{DA}-\lambda_D > 0$, $\frac{\partial \beta_D}{\partial S_{DB}} > 0$ and $\frac{\partial \lambda_D}{\partial S_{DB}} > 0$. (To see that $\lambda_{DA}-\lambda_D > 0$, refer to equation (21) and perform the same analysis as in lemma 5.) Thus, the downward sloping curve in a diagram like in figure 1 shifts upward with S_{DB}, implying that λ_{DA} and λ_{DB} depend positively on S_{DB}.

We now show that S_{DB} is uniquely defined by equation (25). Considering that $\lambda_{0B} = \frac{S_{0B}}{N_0}$, we can rewrite equation (25) as follows: $\frac{S_B - S_{DB}}{N_0} = \ln\left(\frac{\lambda_{DB}e^{\lambda_{DA}}}{1-e^{-\lambda_{DB}}}\right)$. Thus, the left hand side of equation (25) depends negatively on S_{DB}, while the right hand side of equation (25) depends positively on S_{DB}:

$$\frac{\partial}{\partial S_{DB}}\left(\ln\left(\frac{\lambda_{DB}e^{\lambda_{DA}}}{1-e^{-\lambda_{DB}}}\right)\right) = \frac{\left[1-e^{-\lambda_{DB}}\left(1+\lambda_{DB}\right)\right]\frac{\partial \lambda_{DB}}{\partial S_{DB}} + \lambda_{DB}\left(1-e^{-\lambda_{DB}}\right)\frac{\partial \lambda_{DA}}{\partial S_{DB}}}{\lambda_{DB}\left(1-e^{-\lambda_{DB}}\right)} > 0.$$

Thus, equation (25) defines a unique value of S_{DB}.

The number of successful matches can then be expressed as: $M_{0B} = S_{0B}\frac{1-e^{-\lambda_{0B}}}{\lambda_{0B}}$, $M_{DA} = S_A\frac{1-e^{-\lambda_{DA}}}{\lambda_{DA}}$ and $M_{DB} = S_{DB}\frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}} = \left(S_B - N_0\lambda_{0B}\right)\frac{1-e^{-\lambda_{DB}}}{\lambda_{DB}}$. Given $M_0 = M_{DA} + M_{DB} + M_{0B}$ we can then solve for $I = L - S + \left[V_{DA} + \left(1 - \frac{M_{DA}}{S_A}\right)\right]S_A + \left[V_{DB} + \left(1-\frac{M_{DB}}{S_{DB}}\right)\right]S_{DB} + \left[V_{0B} + \left(1-\frac{M_{0B}}{S_{0B}}\right)\right]S_{0B}$

$+M_{DA}\left(\pi_0 - b_{DA}\right) + M_{DB}\left(\pi_0 - b_{DB}\right) + M_{0B}\left(\pi_0 - b_{0B}\right) = L + M_0\left(\pi_0 - 1\right)$, which in turn yields $q_0 = \alpha\frac{\sigma-1}{\sigma-\alpha}\left[\frac{L}{M_0} - (1+f)\right]$ and $\pi_0 = \frac{1}{\sigma-\alpha}\left(\alpha\left[\frac{L}{M_0}-1\right] - \sigma f\right)$, which the allows us to solve for b_{0B}, b_{DA}, b_{DB} and the corresponding expected payments.

We now show that $\lambda_{DA} < \lambda_A$. To see this fact suppose instead that $\lambda_{DA} \geq \lambda_A$, which implies that $N_{DA} \leq N_A$ (because S_A cannot decrease). But then $N_{DB} \geq N_B$ and, therefore, $\lambda_{DB} < \lambda_B$ so that $E\left(\pi^{net}_{DA}\right) > E\left(\pi^{net}_{DB}\right)$ which does not satisfy equation (20). Hence, $\lambda_{DA} < \lambda_A$. Finally, to see that

$\lambda_{DB} < \lambda_B$ note from equation (22) that λ_{DB} is increasing in β_D and in λ_D. This result is similar to that in proposition 4. Hence, because some of the B-labels apply to the non-discriminatory firms we must have $\beta_D < \beta$ and because $S_{DA} = S_A$ we must also have $\lambda_D < \lambda_U$. Hence, $\lambda_{DB} < \lambda_B$. □

An additional facet of the equilibrium with N_0 non-discriminatory firms is that holding π constant we have $V_{0B} = V_{DB} > V_B$ and $V_{DA} > V_A$ as seen in figure 5. To see the first point consider equation (23) and note that $\lambda_{DB} < \lambda_B$ and that $\lambda_{DA} < \lambda_A$. To see the second note that $V_{DA} = \pi e^{-\lambda_{DA}}$ which is decreasing in λ_{DA}. In addition, using equation (24) and the left hand side of (23) yields that $V_{0B} = \pi_0 e^{-\lambda_{0B}}$ which combined with the fact that $V_{0B} = V_{DB} < V_{DA}$ yields that $\lambda_{0B} > \lambda_{DA} > \lambda_{DB}$ as seen in figure 5. Similarly, holding π constant and noting that $E\left(\pi_{DA}^{net}\right) = E\left(\pi_{DB}^{net}\right)$, that $E\left(\pi_{DA}^{net}\right)$ is increasing in λ, and that $\lambda_{DA} < \lambda_A$ we see that expected firm profits of the discriminatory firms are lower in the coexistence equilibrium than in the fully discriminatory equilibrium without any label-blind firms.

In order to analyze the full effect of the non-discriminatory firms on expected bonuses and expected profits we, therefore, also need to consider how realized firm profits are affected. As in the previous cases realized firm profits in the coexistence equilibrium, π_0, are a function of the number of successful matches, M_0, as well as several exogenous variables that do not depend on the particular equilibrium under consideration. We, therefore, now consider the vacancy rate in the equilibrium with N_D discriminatory and N_0 label-blind firms. The average vacancy rate in this equilibrium can be written as:

$$\Psi_0 = \frac{N_{DA}}{N} e^{-\lambda_{DA}} + \frac{N_{DB}}{N} e^{-\lambda_{DB}} + \frac{N_{0B}}{N} e^{-\lambda_{0B}} = \eta_A e^{-\frac{S_A}{\eta_A N}} + (1 - \eta_A - \eta_0)\, e^{-\frac{S_{DB}}{(1-\eta_A-\eta_0)N}} + \eta_0 e^{-\frac{S_{0B}}{\eta_0 N}}, \quad (26)$$

where $\eta_A = \frac{N_{DA}}{N}$ and $\eta_0 = \frac{N_0}{N}$. The partial derivative of this vacancy rate with respect to the portion of non-discriminatory firms is:

$$\frac{\partial \Psi_0}{\partial \eta_0} = (1 + \lambda_{0B}) e^{-\lambda_{0B}} - (1 + \lambda_{DB}) e^{-\lambda_{DB}} < 0. \quad (27)$$

To see that equation (27) is strictly negative note that $(1 + \lambda) e^{-\lambda}$ is strictly decreasing in λ and remember from proposition 12 that $\lambda_{0B} > \lambda_{DB}$. Hence, an increase in the portion of non-discriminatory firms yields a greater number of matches which in turn reduces the realized firm profits of successful firms. This reduced profit reinforces the negative effect on discriminatory firm profits discussed in the previous paragraph but renders ambiguous the effect on the skilled workers' expected payoff.

We state the results of the previous two paragraphs as proposition 13.

Proposition 13. *In the unique equilibrium with $N_0 < \beta N$ non-discriminating firms and $N_D = N - N_0$ discriminating firms the expected number of successful matches is increasing in the number of label-blind firms. Expected firm profits of the discriminatory firms are lower in the coexistence equilibrium and they are decreasing in N_0.*

An important implication of proposition 13 is that if there are two economies that differ only in the portion of label-blind firms (while holding the total number of firms constant), then the country with more discriminatory firms would have a comparative advantage in the numeraire sector. We analyze this implication in the following section.

8 Can trade ameliorate discrimination?

We start by analyzing the pattern of trade in our augmented model where label-blind firms coexist with discriminatory ones. Remembering that $p_z = \frac{\sigma}{\sigma-1}$, we can rewrite equation (16) as:

$$\frac{P_M}{P_0} = \frac{\alpha}{1-\alpha}\frac{C_0}{C_M} = \frac{M^{\frac{1}{1-\sigma}}\sigma}{\sigma - 1} > \frac{(M^*)^{\frac{1}{1-\sigma}}\,\sigma}{\sigma - 1} = \left(\frac{P_M}{P_0}\right)^*. \tag{28}$$

Analysis of equation (28) reveals that the only determinant of comparative is the expected number of matches. Hence, if the home and foreign countries only differ in the proportion of label-blind firms, we can then say that the more discriminatory, or more nepotistic, country is the one that has a smaller number of label-blind firms. A natural corollary of propositions 8 and 13 is then that comparative advantage can be determined solely from the relative proportions of label-blind firms in each country.

Corollary 14. *If the home country has N_D discriminatory and $N_0 < \beta N$ non-discriminatory firms, the foreign country has N_D^* discriminatory and $N_0^* < \beta N^*$ label-blind firms, the total number of firms is the same, $N = N^*$, and technology is the same in both countries, then the country with more discriminatory firms has a comparative advantage in the numeraire sector.*

From corollary 14 we can then say that a greater degree of nepotism or discrimination can cause a country to become an exporter of simpler products and a net importer of products from the more sophisticated manufacturing sector.

The introduction of some non-discriminatory firms also allows us to consider the effect of trade liberalization on the prevalence of discrimination. In particular, we analyze how the movement from autarky to free trade affects the expected profits of discriminatory and non-discriminatory firms. The important difference between the two types of firms is that the label-blind firms have larger expected profits. The realized profits of all successful firms in the coexistence equilibrium with trade, π_0^{trade}, is the same, however, a label-blind firm receives a greater proportion of that profit in expectation. Hence, the effect of trade liberalization on realized profits has a larger, magnified, effect on the expected profits of non-discriminatory firms.

Proposition 15. *In the movement from autarky to free trade the expected profits of the label-blind firms will change by more than than those of the discriminatory firms. Hence, trade liberalization will disproportionately affect the non-discriminatory firms.*

Proposition 15 suggests that trade liberalization will make it more costly to discriminate in countries where there are fewer discriminatory firms and less costly where it already more prevalent. In this way trade liberalization will magnify the good and the bad institutions that a country has in autarky.

Propositions 12 and 15 together provide some support and some limitations of the suggestion in Becker (1957) and Arrow (1972) that the market can ameliorate discrimination. First, proposition 12 shows that non-discriminatory firms earn larger expected profits (the extra cost that discriminatory firms pay for their preferences are in the form of a reduced matching rate), which provides some support for Becker's hypothesis. On the other hand, proposition 15 shows that trade liberalization can reinforce a country's market imperfections (and perfections) and affect the expected profits of label-blind firms by more than those of discriminatory firms.

9 Conclusion

We embed a competitive search model with labor market discrimination, or nepotism, into a two-sector, two-country framework in order to analyze the relationship between international trade and labor market discrimination. Discrimination reduces the matching probability and output in the skilled-labor intensive differentiated-product sector so that the country with more discriminatory firms has a comparative advantage in the simple sector. As countries alter their production mix in accordance with their comparative advantage, trade liberalization can then reinforce the negative effect of discrimination on development in the more discriminatory country and reduce its effect in the country with fewer discriminatory firms. Similarly, the relative profit difference between non-discriminatory and discriminatory firms will increase in the less discriminatory country and shrink in the more discriminatory one. In this way trade can further reduce discrimination in a country where it is less prevalent and increase it where it is more firmly entrenched.

Appendix

Totally differentiating $V_B = b_z e^{-\lambda_A} \frac{1-e^{-\lambda_B}}{\lambda_B}$ and $V_A = b_z \frac{1-e^{-\lambda_A}}{\lambda_A}$, considering that V_B and V_A are constant from a single firm's perspective and solving for $\frac{d\lambda_B}{db}$ and $\frac{d\lambda_A}{db}$ leads to:

$$\frac{d\lambda_A}{db} = -\frac{\left(1 - e^{-\lambda_A}\right)\lambda_A}{b\left(e^{-\lambda_A}\lambda_A - 1 + e^{-\lambda_A}\right)}$$

$$\frac{d\lambda_B}{db} = -\frac{\lambda_B\left(1 - e^{-\lambda_B}\right)\left(\lambda_A - 1 + e^{-\lambda_A}\right)}{\left(e^{-\lambda_B}\lambda_B - 1 + e^{-\lambda_B}\right) b\left(e^{-\lambda_A}\lambda_A - 1 + e^{-\lambda_A}\right)}.$$

Thus, we get:

$$\frac{d\lambda_A}{db} + \frac{d\lambda_B}{db} = -\frac{1}{b}\frac{\lambda_A\left(e^{\lambda_A} - 1\right)}{\left(e^{\lambda_A} - \lambda_A - 1\right)}\left[\frac{\lambda_B\left(e^{\lambda_B} - 1\right)}{2\left(e^{\lambda_B} - \lambda_B - 1\right)}\frac{2\left(\lambda_A e^{\lambda_A} - e^{\lambda_A} + 1\right)}{\lambda_A\left(e^{\lambda_A} - 1\right)} - 1\right].$$

$\frac{d\lambda_A}{db} + \frac{d\lambda_B}{db} < 0$ since $e^{\lambda_A} - \lambda_A - 1 > 0$, $\frac{\lambda_B(e^{\lambda_B}-1)}{2(e^{\lambda_B}-\lambda_B-1)} > 1$ and $\frac{2(\lambda_A e^{\lambda_A} - e^{\lambda_A}+1)}{\lambda_A(e^{\lambda_A}-1)} > 1$.

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