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Optimal Taxation of Capital Income in Models with Endogenous Fertility

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Abstract

This paper studies the issue of the efficient taxation of capital income in intertemporal optimizing models with infinite horizons and endogenous population growth. We discover that, in the steady state, the optimal capital income tax is negative when the economy is closed. Instead, in a small open economy facing perfect capital mobility, the Chamley-Judd result of a zero tax rate is obtained if capital taxation is source-based; otherwise, income from wealth should be subsidized if taxation is residence-based. Moreover, we find that in our setup, taxing capital income with immediate expensing of capital expenditure may replicate the first-best equilibrium when labor is subsidized. Our findings, which depart substantially from those obtained in representative agent models with an endogenous labor supply, are to be ascribed to a wealth effect in the fertility choices that directly affects the pseudo-welfare function of the social planner.

JEL classification: E62; H22; J22; O41.

Keywords: Factor Income Taxes; Second-best Analysis; Endogenous Population Growth; Capital Formation.

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1 Introduction

A recurrent question vivifies the doctrinary and policy debate regarding the desiderable tax structure: given the highly distortionary nature of capital taxes, should capital income be alleviated from the burden of taxation? See, for example, Boskin (1996 and 2005), Altig *et alii* (2001), Slemrod and Bakija (2004), and the *2005 Economic Report of the President*'s Tax Chapter.

The idea of a zero capital income tax rate, largely in the background of this debate, originates from Judd (1985) and Chamley (1986), who address the question of factor income taxation by using an intertemporal second-best perspective. Judd (1985) analyzes the redistributive potential of a capital income tax in a neoclassical growth model with Kaldorian heterogeneity where capital tax revenues are lump-sum transferred from capitalists to workers. He discovers, surprisingly, that setting capital income taxes equal to zero in the long-run is optimal also from the workers' standpoint; this is because a capital levy depresses workers' disposable income, as the long-run supply of capital is infinitely elastic. In a representative agent economy with labor endogenously supplied, Chamley (1986) argues that the optimal dynamic tax configuration is one in which capital income should be exempted from taxation, while labor income should bear the tax burden required to finance a given stream of government expenditure.[1] The Chamley discovery is only

[1]As evidenced by Chari and Kehoe (1999), and Judd (1999), the intuitive foundation for the optimal zero tax on capital income can be derived from two classic principles of commodity tax theory: i) intermediate goods should be exempted from taxation as taxes are to be levied only on final goods (Diamond and Mirrlees, 1971); and ii) all commodities should be taxed at a uniform rate (Atkinson and Stiglitz, 1972). While a labor income tax solely distorts the static consumption-leisure decisions, a tax on capital income generates an intertemporal distortion between the marginal rate of substitution of consumption at two different dates and the corresponding marginal rate of transformation; such a distortion increases exponentially in time. Therefore, taxing capital income entails taxing consumption at different dates differently, thus violating the normative principle of uniform taxation of consumption goods. Moreover, productive efficiency requires that the capital stock (an intermediate good since it does not enter the utility function) should

valid asymptotically. If the instantaneous utility function of consumers is strongly separable in consumption and leisure, and in addition is isoelastic in consumption, welfare maximization implies that capital should be taxed at 100 percent at the initial period and zero afterwards; this is because the capital stock is inelastic in the short-run and therefore it must be efficiently taxed at a confiscatory rate.

Lucas (1990), Jones, Manuelli and Rossi (1993 and 1997), Correia (1996b), Atkeson, Chari and Kehoe (1999), and Judd (1999), among others, find that the optimality of the zero capital income tax carries over a wide variety of setups that incorporate human capital accumulation, perpetual growth, perfect capital mobility and overlapping-generations.[2]

The second-best principle of capital taxation established by Judd (1985) and Chamley (1986), however, is not an ineluctable law of dynamic public finance as shown, for example, by Correia (1996a), Jones, Manuelli and Rossi (1997), Chamley (2001), Erosa and Gervais (2002), and Abel (2006).[3] Correia (1996a) discovers that the introduction of an additional factor of production, which cannot be optimally taxed or subsidized, in a Ramsey-Ricardo exogenous growth model leads to a violation of the zero capital tax prescription. From a methodological perspective, Jones, Manuelli and Rossi (1997) identify two types of changes in the neoclassical intertemporal framework that lead to a capital income tax different from zero. The first change is obtained if the capital stock appears in the pseudo-welfare function of the social planner; this case is satisfied, for example, when the labor supply is inelastic (i.e. pure rents enter the consumer's budget constraint) and there

be untaxed, while labor (a final good as it appears in the utility function of consumers through leisure) should be taxed.

[2] In particular, Atkeson, Chari and Kehoe (1999) provide a generality test of the Chamley-Judd tax result by systematically relaxing one by one the hypotheses that support it and discovering that, in so doing, its validity remains unaffected.

[3] Other contributions in which optimal capital income taxation may differ from zero are by Pestieau (1974), Atkinson and Sandmo (1980), Lansing (1999), Boadway, Marchand and Pestieau (2000), and Cremer, Pestieau, and Rochet (2003).

is a bound on labor income taxation.[4] The second change is represented by the case in which the planner faces different constraints from the consumer involving the capital stock; an example of this circumstance occurs when the planner cannot distinguish between the income from two types of labor and there are restrictions on the tax codes (namely, the two types of labor should be equally taxed).

Erosa and Gervais (2002) provide another example in which the Chamley-Judd result is inapplicable. They use a standard life-cycle growth model, in which, as individuals have a labor-leisure choice in each period of their lives, and hence the individual's optimal consumption-work plan is almost never constant, it is optimal almost always to tax consumption goods and labor earnings at different rates over an individual's lifetime. This goal can be achieved by using capital and labor income taxes that vary with age. Judd (2002), instead, obtains a negative capital income tax rate in a dynamic model with monopolistic competition in the product market; in his view, a capital subsidy can lighten the allocative inefficiency due to the monopoly power of firms, thereby contrasting the reduction of capital and output implied by the non-competitive equilibrium. By using an infinitely-lived model of capital formation, Abel (2006) shows that if capital expenses could be deducted immediately from taxable capital income, tax efficiency would prescribe a positive tax on capital income and a zero tax on labor income. This optimal tax structure, which is the opposite of the Chamley-Judd principle, exactly replicates the first-best allocation obtainable with lump-sum taxation.

In this paper, we study the issue of the optimal capital income taxation in neoclassical growth models with infinite horizons and elastic fertility choices, i.e. endogenous population growth; the analysis considers closed and small open economies. The hypothesis of endogenous fertility has been neglected by the copious literature on efficient capital income taxation based on in-

[4]The case developed by Correia (1996a) can be viewed as another example of such a change.

tertemporal optimizing models. One possible explanation for this lack of analysis is the apparent similarity of infinitely-lived models based on endogenous fertility with those incorporating elastic labor-leisure choices. From a normative perspective on factor income taxation, this paper demonstrates that a model with endogenous fertility is qualitatively non-equivalent to a model with an elastic labor supply as their relative prescriptions on capital income taxation differ substantially.

We discover that, in a closed economy, the optimal tax configuration prescribes, in the limit, the subsidization of capital income and the taxation of labor income.[5] This result is to be attributed to a wealth effect in the fertility choices; such an effect stems from the fact that the opportunity cost of fertility, which in equilibrium must be equal to the marginal rate of substitution of consumption for fertility, is given by the wage cost of child rearing plus the stock of wealth (namely, the capital stock).

If an open economy facing perfect capital mobility is taken into account, two possible regimes for taxing capital income should be considered: the residence-based or worldwide regime and the source-based or territorial regime. According to the residence-based principle, the tax is levied on capital income of domestic residents regardless of the country where income is originated; residence-based taxes represent taxes that accrue on nonhuman wealth. Source-based capital income taxes, instead, are imposed on all capital income obtained in a particular jurisdiction regardless of the residence country of savers; in this case, the location of capital, i.e. investment, is subject to taxation.

When a financially integrated small open economy is considered, an endogenous population growth implies that the efficient capital income taxation strictly depends on which regime is adopted. Differently from Correia (1996b), Atkeson, Chari and Kehoe (1999), and Chari and Kehoe (1999),

[5] The result of a negative capital income tax rate is not new, having also been obtained by Correia (1996a), when the additional untaxed factor of production is Edgeworth substitutable with capital, and Judd (2002).

where the Chamley-Judd principle is confirmed under both international regimes of wealth income taxation, our key findings involve either subsidizing capital, if taxation is residence-based, or avoiding the distortion of capital formation, if taxation responds to a source-based criterion; a fiscal levy on labor income, irrespective of the taxation regime, should be imposed.

Finally, we examine, in the context of endogenous fertility, the proposal of capital income taxation combined with an immediate expensing of capital expenditure as proposed by Hall and Jorgenson (1967), and studied in a general equilibrium context by Abel (2006). In a closed economy, the first-best equilibrium is replicated if the tax on capital is accompanied by a labor subsidy. In the open economy analysis, the closed economy findings are reiterated if gross saving is expensed from taxable wealth income when the residence-based regime is adopted. Our results stand in sharp contrast with those obtained by Abel (2006) in a closed economy, where efficiency requires to tax capital and free labor from taxation.

The paper is structured as follows. Section 2 presents the closed economy model with endogenous fertility choices, and analyzes its positive and normative implications for factor income taxation. Section 3 extends the previous analysis to a small open economy facing perfect capital mobility. Section 4 discusses the proposal of expensing capital expenditure from taxable income in the case of endogenous population growth. Section 5 concludes.

2 Closed economy

2.1 The setup

Consider a real closed economy peopled by immortal consumers, who decide on consumption, fertility, and saving on an intertemporal basis.[6] The representative consumer of this economy maximizes the following integral

[6]The model employed here builds on Razin and Ben-Zion (1973), Palivos (1995) and Barro and Sala-i-Martin (2003, ch. 9).

utility

$$\int_0^\infty U(c,n)\mathrm{e}^{-\rho t}dt, \tag{1}$$

where c is per capita consumption, n the fertility rate, and ρ the exogenous rate of time preference.[7] The instantaneous utility function $U(c,n)$, which is strictly increasing in c and n, satisfies the usual properties of regularity. c and n are assumed to be normal goods.

Two constraints must be respected when (1) is maximized. One is the intertemporal budget constraint, given by

$$\int_0^\infty [c-(1-\tau_l)wl-q]\mathrm{e}^{-\int_0^t[(1-\tau_k)(r-\delta)-n]ds}dt = k_0, \tag{2}$$

where w represents the wage rate, l labor hours, q lump-sum transfers from the government (in per capita terms), r the before-tax interest rate, δ the constant capital depreciation rate, and k the per capita capital stock (k_0 is k at time 0). τ_k and τ_l indicate *ad valorem* capital and labor income tax rates, respectively; capital depreciation allowances are permitted.

Moreover, the time allocation constraint

$$l + T(n) = 1, \tag{3}$$

must also be considered in addition to (2) when (1) is maximized. According to (3), the fixed time endowment (normalized to one) can be used either for working or for raising children. $T(\,\cdot\,)$ denotes the amount of time devoted to child-rearing (with $T' > 0$ and $T'' \lesseqgtr 0$).

The maximization of (1) subject to (2) and (3) yields the following first-order conditions

[7] Note that n also represents the population growth rate as the mortality rate is zero and the economy is closed.

$$U_c \mathrm{e}^{-\rho t} = \lambda \mathrm{e}^{-\int_0^t [(1-\tau_k)(r-\delta)-n]ds}, \tag{4a}$$

$$U_n \mathrm{e}^{-\rho t} = \lambda \mathrm{e}^{-\int_0^t [(1-\tau_k)(r-\delta)-n]ds} [(1-\tau_l)wT' + k_0 \mathrm{e}^{\int_0^t [(1-\tau_k)(r-\delta)-n]ds}], \tag{4b}$$

where $\lambda = U_c[c(0), n(0)] = U_c[0]$ is the Lagrange multiplier on the intertemporal budget constraint (2).

Equation (4b) can be rewritten as

$$\frac{U_n}{U_c} = (1-\tau_l)wT' + k, \tag{4b'}$$

while from (4a), we obtain the following Euler equation

$$-\frac{d}{dt} \ln U_c = (1-\tau_k)(r-\delta) - n - \rho. \tag{4c}$$

Equation (4b') asserts that the marginal rate of substitution of consumption for fertility must equal the opportunity cost of one unit of fertility, given by the after-tax wage times the marginal time-cost of child-rearing plus the per capita capital stock. Equation (4c) ensures that in the intertemporal equilibrium the rate of return on consumption, i.e. $\rho - \frac{d}{dt} \ln U_c$, is equal to the after-tax return on per capita capital, namely, $(1-\tau_k)(r-\delta) - n$.

Production is carried out by many competitive firms. The production function, which is given by $y = F(k, l)$, satisfies the usual neoclassical properties of regularity and is linearly homogeneous in k and l.

Maximum profits requires

$$F_k(k, l) = r, \tag{5a}$$

$$F_l(k, l) = w. \tag{5b}$$

The resource constraint is given by

$$F(k, l) = c + \dot{k} + (\delta + n)k + g. \tag{6}$$

Finally, the government balances its budget by financing public expenditures through factor income taxation

$$\tau_k(r - \delta)k + \tau_l wl = g + q, \tag{7}$$

where g denotes the exogenous per capita government consumption expenditure.

2.2 Positive analysis

In the steady state, the macroeconomic model can be succintly written as[8]

$$\frac{U_n(c, n)}{U_c(c, n)} = (1 - \tau_l)F_l[k, 1 - T(n)]T'(n) + k, \tag{8a}$$

$$(1 - \tau_k)\{F_k[k, 1 - T(n)] - \delta\} = \rho + n, \tag{8b}$$

$$F[k, 1 - T(n)] = c + (\delta + n)k + g. \tag{8c}$$

We assume that lump-sum transfers q adjust endogenously to maintain the government budget in equilibrium, while g is given.

[8] The dyamic properties of the model are studied in Palivos (1995). The model exhibits saddle-point stability if the condition $T'^2 F_{ll} - F_l T'' < 0$ is satisfied.

The above equations may be written in implicit form as[9]

$$n = n(c, k; \tau_l), \quad n_c > 0;\ n_k < 0;\ n_{\tau_l} > 0; \tag{8a'}$$

$$k = k(n; \tau_k), \quad k_n < 0;\ k_{\tau_k} < 0; \tag{8b'}$$

$$c = c(k, n), \quad c_k > 0;\ c_n < 0. \tag{8c'}$$

By reducing the after-tax wage and hence the opportunity cost of fertility, an increase in τ_l stimulates the fertility rate (for a given level of consumption).[10] Hence, more time for child rearing is required; a reduction in labor hours worked takes place. The capital stock is driven down because there is an inverse relationship between per capita capital and population growth; this is a sort of reverse Malthus law implied by the "modified golden rule" (8b). Consumption is reduced by the lower capital stock and the higher fertility rate.

A rise in τ_k pulls the capital stock down as, for a given fertility rate, the after-tax marginal product of capital is lowered.[11] The fall in k generates two contrasting effects on fertility; it stimulates the demand for fertility as the

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[9] The expressions of the partial derivatives of relationships (8') are
$n_c = \dfrac{U_n U_{cc} - U_c U_{cn}}{U_c \Lambda} > 0;\ n_k = \dfrac{U_c[(1-\tau_l)T' F_{lk} + 1]}{\Lambda} < 0;\ n_{\tau_l} = -\dfrac{U_c T' F_l}{\Lambda} > 0;$
$k_n = \dfrac{[1 + (1-\tau_k)T' F_{kl}]}{(1-\tau_k)F_{kk}} < 0;\ k_{\tau_k} = \dfrac{(F_k - \delta)}{(1-\tau_k)F_{kk}} < 0;$
$c_k = F_k - \delta - n > 0;\ c_n = -(F_l T' + k) < 0;$
where $\Lambda = U_{nn} - \dfrac{U_n}{U_c} U_{cn} + (1-\tau_l)U_c(T'^2 F_{ll} - F_l T'') < 0.$

[10] The basic steady state effects of labor taxation are given by:
$\dfrac{dn}{d\tau_l} = \dfrac{n_{\tau_l}}{\Omega} > 0;\ \dfrac{dk}{d\tau_l} = \dfrac{k_n n_{\tau_l}}{\Omega} < 0;\quad \dfrac{dc}{d\tau_l} = \dfrac{(c_k k_n + c_n) n_{\tau_l}}{\Omega} < 0;$
where $\Omega = 1 - k_n(n_k + n_c c_k) - c_n n_c > 0$.

[11] A rise in τ_k has the following long-run effects
$\dfrac{dn}{d\tau_k} = \dfrac{(n_c c_k + n_k) k_{\tau_k}}{\Omega} \lesseqgtr 0;\quad \dfrac{dk}{d\tau_k} = \dfrac{(1 - c_n n_c) k_{\tau_k}}{\Omega} < 0;\quad \dfrac{dc}{d\tau_k} = \dfrac{(c_k + n_k c_n) k_{\tau_k}}{\Omega} < 0;$
where $\Omega > 0$ has been defined in footnote 10.

opportunity cost of fertility falls, on the one side, and discourages fertility because of the reduction in the after-tax marginal productivity of capital, on the other. Therefore, fertility moves unclearly, while consumption is unambiguously reduced.

2.3 Normative analysis

The problem of efficient taxation, known as the "Ramsey problem", is studied by using the so-called 'primal method' in the version developed by Lucas and Stokey (1983); such a method is based on the concept of implementability constraint, which is obtained from the households' intertemporal budget constraint by expressing prices and taxes in terms of quantities through the marginal conditions (4).[12] Optimal taxation is analyzed under the assumption that total government spending is fixed.

Plugging (3), (4a), (4b') and $\lambda = U_c[0]$ into (2), we get the implementability constraint, given by

$$\int_0^{\infty} [(c-q)U_c - \frac{(1-T)}{T'}(U_n - kU_c)]\mathrm{e}^{-\rho t}dt = k_0 U_c[0]. \tag{9}$$

The efficient taxation of factor income is found by maximizing the utility functional (1) subject to the implementability constraint (9) and the feasibility constraint (6), once the time allocation constraint (3) is brought in.

Define the pseudo-welfare function as

$$W(c,n,k,\Phi) = U(c,n) + \Phi\{(c-q)U_c(c,n) - \frac{[1-T(n)]}{T'(n)}[U_n(c,n) - kU_c(c,n)]\},$$

where Φ is the Lagrange multiplier associated with (9). Φ is positive in the case of distortionary taxation of labor income.

The second-best problem can be formulated in a formal way as follows:

[12] See Lucas (1990), and Chari and Kehoe (1999) for an application of such a methodology to the problem of optimal capital taxation.

$$\max \int_0^\infty W(c,n,k,\Phi)\mathrm{e}^{-\rho t}dt \tag{10a}$$

subject to

$$\dot{k} = F[k, 1-T(n)] - c - (\delta+n)k - g. \tag{10b}$$

We show that:

Proposition 1 *In a closed economy model of capital accumulation with endogenous population growth and infinitely-lived consumers, tax efficiency requires the subsidization of capital in the long-run; this implies that it is optimal to tax labor income in order to finance a given stream of government spending and the capital subsidy.*

Proof. The first-order conditions for the "Ramsey optimum" (10) are

$$W_c \mathrm{e}^{-\rho t} = \Gamma, \tag{11a}$$

$$W_n \mathrm{e}^{-\rho t} = \Gamma(F_l T' + k), \tag{11b}$$

$$\dot{\Gamma} = -W_k \mathrm{e}^{-\rho t} - \Gamma(F_k - \delta - n), \tag{11c}$$

where Γ is the co-state variable on the feasibility constraint, $W_c = U_c[1+\Phi(1+\eta_c)]$, $W_n = U_n[1+\Phi(1+\eta_n)]$, and $W_k = \Phi\dfrac{(1-T)}{T'}U_c$. η_c and η_n represent general equilibrium elasticities for consumption and fertility, respectively.[13]

[13] These elasticities are defined as $\eta_c = (c-q)\dfrac{U_{cc}}{U_c} - \dfrac{(1-T)}{T'}\dfrac{(U_{nc}-kU_{cc})}{U_c}$;
$\eta_n = (c-q)\dfrac{U_{cn}}{U_n} + \dfrac{(1-T)T''}{T'^2} - k\dfrac{U_c}{U_n}[1+\dfrac{(1-T)T''}{T'^2}] - \dfrac{(1-T)}{T'}\dfrac{(U_{nn}-kU_{nc})}{U_n}$.

Equation (11c) can be rewritten with the aid of (11a) as

$$-\frac{d}{dt}\ln W_c = \frac{W_k}{W_c} + F_k - \delta - n - \rho. \tag{11c'}$$

In the steady state, (4c) and (5a) imply that $(1-\tau_k)(F_k-\delta) = \rho + n$. Combining this equation with the long-run version of (11c'), one gets the optimal capital tax rate τ_k^*; that is,

$$\tau_k^* = -\frac{W_k}{(F_k-\delta)W_c} < 0.$$

Normatively speaking, physical capital should be subsidized in the long-run. The efficient labor income tax rate, obtained by the government budget constraint after using τ_k^*, should instead be positive. □

Our long-run results are to be ascribed to the fact that per capita capital enters the demand for fertility. This wealth effect is obtained because the opportunity cost of fertility depends on nonhuman wealth as population growth erodes its stock in per capita terms. Since the static efficiency condition for fertility enters the implementability constraint (9) and hence the pseudo-welfare function of the social planner, the capital stock appears directly in the maximand function of the "Ramsey problem", thus altering the Chamley-Judd optimal capital tax rule.[14]

The *ratio* of this optimal tax configuration is imputable to the fact that -when labor is taxed and hence fertility is stimulated and capital diminished- capital subsidization becomes necessary in order to dampen population growth and increase capital formation, thus allowing the economy to attain a higher level of consumption and welfare of consumers.

[14] This is consistent with the methodological remarks on the zero capital tax rule put forward by Jones, Manuelli and Rossi (1997, p. 105-06).

3 Small open economy

3.1 The model

This section investigates the implications of endogenous fertility choices on the efficient capital income taxation in an international context. Let us extend the previous analysis to a small open economy that produces a single tradable good, perfectly substitutable with the foreign produced good, and that operates in a perfectly globalized capital market. Perfect capital mobility implies that the relevant domestic interest rate is determined by the exogenous world interest rate, denoted by r^*.[15] Moreover, we assume that no international migration of workers occurs.

Financial wealth per capita, a, consists of physical capital and net foreign assets per capita, which are denoted by b; that is, $a = k + b$. While b may be either positive or negative, a is considered to be strictly positive.

There are two systems of taxing income from capital in an open economy: the residence-based (often called worldwide) system and the source-based (also known as territorial) system.

Under the *residence-based system of capital taxation*, incomes from domestic and foreign wealth are taxed uniformly. Therefore, perfect capital mobility requires that the before-tax rates of return on each asset are equalized; that is, $r - \delta = r^*$. The households' flow budget constraint is given by

$$\dot{k} + \dot{b} = [(1 - \tau_a)r^* - n](k + b) + (1 - \tau_l)wl + q - c, \tag{12}$$

where τ_a represents a proportional tax rate on income from financial wealth inspired by the residence-based principle of taxation.

The maximization of (1) subject to (3) and (12) yields

$$\frac{U_n}{U_c} = (1 - \tau_l)wT' + k + b, \tag{13a}$$

[15] It is assumed that saving is not taxed abroad.

$$-\frac{d}{dt}\ln U_c = (1-\tau_a)r^* - n - \rho. \tag{13b}$$

From (13a), it is clear that in an open economy (relatively to a closed one) the opportunity cost of one unit of fertility is raised by b. Since r is pinned down by $r^* + \delta$, the input demand system (5) implies that the capital intensity and the wage rate are fixed and tax invariant; that is, $\frac{k}{l} = \kappa^*$ and $w = w^*$, where $\kappa^* = f'^{-1}(r^* + \delta) > 0$, $w^* = f(\kappa^*) - \kappa^* f'(\kappa^*)$ and $f(\frac{k}{l}) = F(\frac{k}{l}, 1)$ is the output-labor ratio.

Alternatively, in the case of *source-based capital taxation*, the arbitrage condition between domestic and foreign assets requires that the after-tax return on capital is equal to the world interest rate, i.e. $(1-\tau_k)(r-\delta) = r^*$, where τ_k represents the domestic capital income tax rate. Therefore, the representative agent's dynamic budget constraint becomes

$$\dot{k} + \dot{b} = (r^* - n)(k+b) + (1-\tau_l)wl + q - c. \tag{14}$$

From the consumers' standpoint, optimality requires that (13a) and the following Euler equation

$$-\frac{d}{dt}\ln U_c = r^* - n - \rho, \tag{15}$$

are satisfied.

The demand for capital (5a) along with the condition of perfect capital mobility implies that $(1-\tau_k)[F_k(k,l) - \delta] = r^*$. From this relationship, we get that $\frac{k}{l} = \kappa(\tau_k)$ (with $\kappa' = \frac{(f'-\delta)}{(1-\tau_k)f''} < 0$) and, using (5b), $w = w(\tau_k)$ (with $w' = -\frac{\kappa(f'-\delta)}{(1-\tau_k)} < 0$).

In each capital income tax regime, revenues from labor and wealth taxation finance the exogenous stream of government expenditure.

The excess of national income over the aggregate demand gives the rate of accumulation of net foreign assets; in per capita terms, we have

$$\dot{b} = y + (r^* - n)b - c - \dot{k} - (\delta + n)k - g. \qquad (16)$$

3.2 Positive analysis

In this section, we study the comparative static effects of factor tax shifts under the two fiscal regimes.[16]

Residence-based regime

Equation (13b) implies that $n = (1 - \tau_a)r^* - \rho$ in the steady state. Therefore, the long-run model can be expressed as

$$\frac{U_n[c, (1-\tau_a)r^* - \rho]}{U_c[c, (1-\tau_a)r^* - \rho]} = (1 - \tau_l)w^* T'[(1-\tau_a)r^* - \rho] + k + b, \qquad (17a)$$

$$k = \kappa^* \left\{1 - T[(1-\tau_a)r^* - \rho]\right\}, \qquad (17b)$$

$$(\rho + \tau_a r^*)(k + b) + w^* \left\{1 - T[(1-\tau_a)r^* - \rho]\right\} = c + g. \qquad (17c)$$

A rise in τ_l leaves the fertility rate and the capital stock unaffected. The implied reduction in the after-tax wage stimulates the holdings of net foreign assets. An induced rise in disposable income occurs, which in turn raises consumption.

By lowering the after-tax return on wealth, a higher τ_a brings the fertility rate down. As the capital intensity is unchanged, the capital stock is pulled

[16]Note that the hypothesis of endogenous fertility ensures that the dynamics of an infinitely-lived small open economy facing perfect capital mobility are non-degenerate (as occurs in the corresponding setup incorporating elastic labor-leisure choices with adjustment costs associated with capital accumulation), as the economy is saddle-point stable.

up from (17b), since labor hours worked are increased. Consumption and nonhuman wealth unambiguously rise.[17] The stock of net foreign assets may rise or fall. Domestic output and national income are increased.

Source-based regime

As in the long-run $n = r^* - \rho$ from (15), the steady state economy is described by

$$\frac{U_n(c, r^* - \rho)}{U_c(c, r^* - \rho)} = (1 - \tau_l)w(\tau_k)T'(r^* - \rho) + k + b, \tag{18a}$$

$$k = \kappa(\tau_k)[1 - T(r^* - \rho)], \tag{18b}$$

$$[\rho + \frac{\tau_k r^*}{(1 - \tau_k)}]k + \rho b + w(\tau_k)[1 - T(r^* - \rho)] = c + g. \tag{18c}$$

A change in τ_l reproduces the same effects obtained under the residence-based taxation of wealth income. By raising the cost of capital, a hike in τ_k,

[17]The consequences on c and a can be derived as follows. Keeping τ_l fixed, (17a) and (17c) can be written as

$$c = z(a; \tau_a), \;\; z_a > 0; \; z_{\tau_a} < 0; \tag{17a'}$$

$$c = c(a, \tau_a), \;\; c_a > 0; \; c_{\tau_a} > 0; \tag{17c'}$$

where

$z_a = \dfrac{U_c^2}{(U_c U_{cn} - U_n U_{cc})} > 0$; $z_{\tau_a} = -\dfrac{[U_n U_{cn} - U_c U_{nn} + U_c^2(1 - \tau_l)w^* T'']}{(U_c U_{cn} - U_n U_{cc})} < 0$;

$c_a = (\rho + \tau_a r^*) > 0$; $c_{\tau_a} = (a + w^* T')r^* > 0$.

Totally differentiating (17a') and (17c'), the following multipliers are obtained

$\dfrac{dc}{d\tau_a} = \dfrac{(z_a c_{\tau_a} - c_a z_{\tau_a})}{(z_a - c_a)} > 0$; $\dfrac{da}{d\tau_a} = \dfrac{(c_{\tau_a} - z_{\tau_a})}{(z_a - c_a)} > 0$;

where the condition $z_a > c_a$, assumed to be satisfied, ensures saddle-point stability.

instead, lowers the capital intensity and the capital stock, since the fertility rate remains constant. The wage rate falls. Consumption, nonhuman wealth and gross national income are reduced as well.[18] The stock of net foreign assets is, on the contrary, increased.

3.3 Normative analysis

Residence-based regime

In the small open economy, the implementability constraint is given by[19]

$$\int_0^\infty \{(c-q)U_c - \frac{(1-T)}{T'}[U_n - (k+b)U_c]\}\mathrm{e}^{-\rho t}dt = a_0 U_c[0], \qquad (19)$$

[18] To work out the effects on consumption and financial wealth, write (18a) and (18c) as follows

$$c = x(a;\tau_k), \;\; x_a > 0;\; x_{\tau_k} > 0; \qquad (18a')$$

$$c = m(a,\tau_k), \;\; m_a > 0;\; m_{\tau_k} < 0; \qquad (18c')$$

where

$x_a = \dfrac{U_c^2}{(U_c U_{cn} - U_n U_{cc})} > 0;\; x_{\tau_k} = -\dfrac{U_c^2(1-\tau_l)T'w'}{(U_c U_{cn} - U_n U_{cc})} > 0;$

$m_a = \rho > 0;\; m_{\tau_k} = \dfrac{\tau_k r^*(f'-\delta)}{(1-\tau_k)^2 f''} < 0.$

Therefore, the steady state multipliers of consumption and financial wealth -derived from (18')- are

$\dfrac{dc}{d\tau_k} = \dfrac{(x_a m_{\tau_k} - m_a x_{\tau_k})}{(x_a - m_a)} < 0;\; \dfrac{da}{d\tau_k} = \dfrac{(m_{\tau_k} - x_{\tau_k})}{(x_a - m_a)} < 0;$

where saddle-point stability requires $x_a > m_a$.

[19] Equation (19) is obtained by integrating the consumers' flow budget constraint (12) forward, incorporating the condition preventing Ponzi games –that is, $\lim_{t\to\infty} a\, \mathrm{e}^{-\int_0^t[(1-\tau_a)r^*-n]ds}=0$– and using the relationship $U_c\mathrm{e}^{-\rho t} = U_c[0]\mathrm{e}^{-\int_0^t[(1-\tau_a)r^*-n]ds}$. The implementability constraint (19) remains unaltered under the source-based regime of capital taxation even if the relationship (14), the proper "no Ponzi game" condition and the relationship $U_c\mathrm{e}^{-\rho t} = U_c[0]\mathrm{e}^{-\int_0^t(r^*-n)ds}$ are now employed.

while the pseudo-welfare function of the social planner becomes

$$W(c, n, k+b, \Psi) = U(c, n) + \Psi\{(c-q)U_c - \frac{(1-T)}{T'}[U_n - (k+b)U_c]\},$$

where Ψ is the positive Lagrange multiplier on (19).

The optimal tax structure can be obtained by maximizing the present discounted value of the pseudo-welfare function subject to the balance of payments equation (16), once the production function and the time allocation constraint (3) are taken into account.

We can state that:

Proposition 2 *In an immortal small open economy that operates under perfect capital mobility and exhibits endogenous population growth, a residence-based system of taxation implies that the optimal tax rate on wealth is negative in the steady state. Therefore, labor should bear the burden of taxation necessary to finance all government outlays.*

Proof. The optimal social planner problem entails[20]

$$\frac{W_n}{W_c} = F_l T' + k + b, \tag{20a}$$

$$-\frac{d}{dt}\ln W_c = \frac{W_a}{W_c} + F_k - \rho - \delta - n, \tag{20b}$$

$$F_k - \delta = r^*. \tag{20c}$$

[20] The partial derivatives of the pseudo-welfare function are $W_c = U_c[1 + \Psi(1+\varepsilon_c)]$, and $W_n = U_n[1+\Psi(1+\varepsilon_n)]$; ε_c and ε_n are general equilibrium elasticities defined as
$\varepsilon_c = (c-q)\frac{U_{cc}}{U_c} - \frac{(1-T)}{T'}\frac{[U_{nc} - (b+k)U_{cc}]}{U_c}$;
$\varepsilon_n = (c-q)\frac{U_{cn}}{U_n} + \frac{(1-T)T''}{T'^2} - (b+k)\frac{U_c}{U_n}[1 + \frac{(1-T)T''}{T'^2}] - \frac{(1-T)}{T'}\frac{[U_{nn} - (b+k)U_{nc}]}{U_n}$.

In the long-run, (20b) and (20c) imply $r^* = \rho + n - \dfrac{W_a}{W_c}$. As $(1-\tau_a)r^* = \rho + n$ from (13b), we obtain that $\tau_a^* = -\dfrac{W_a}{r^* W_c} < 0$; that is, the optimal tax rate on wealth income under the residence-based system is negative in the steady state. □

By confirming our closed economy findings, we depart from the Chamley-Judd prescription established for a small open economy by Correia (1996b), Atkeson, Chari and Kehoe (1999), and Chari and Kehoe (1999). The rationale for the optimal tax structure just obtained is basically the same as the one highlighted above for the closed economy.

Source-based regime

Under this international tax regime, the social planner optimum implies that relationships (20) must still be satisfied. As the relationships $F_k - \delta = r^*$, from (20c), and $(1-\tau_k)(F_k - \delta) = r^*$, from the condition of perfect capital mobility, must be simultaneously satisfied, it is then requiered that $\tau_k^* = 0$.

This result can be summarized as follows:

Proposition 3 *In a model with endogenous fertility and perfect capital mobility, the optimal source-based taxation of capital income should be asymptotically zero.*

Such a discovery departs from our previous results, but confirms what is found in a small open economy when labor-leisure choices are elastic. The mechanical motivation for this normative result stems from the fact that the before-tax return on the domestic asset must equal the (untaxed) return on net foreign assets. The intuitive motivation is, instead, as follows. As the capital income tax rate does not affect fertility, labor taxation works in a lump-sum fashion. It is then optimal to simply eliminate the tax distortion affecting capital formation to replicate the first-best allocation.

4 Fertility choices and capital taxation with capital expenditure expensing

Abel (2006) investigates the general equilibrium effects of capital income taxation when gross investment is immediately expensed from the tax base as proposed by Hall and Jorgenson (1967). He discovers that, in an infinitely-lived closed economy with an endogenous labor supply the first-best allocation is replicated if the positive capital income tax is accompanied by a zero tax rate on labor income. The non-distortionary nature of this tax structure, which is also valid for the short-run, is due to the exemption of saving from the burden of taxation or, in the Abel (2006) interpretation, to a zero effective capital tax rate.

Here, we study how the Hall and Jorgenson (1967) and Abel (2006) tax proposal works in general equilibrium models with elastic fertility and what its normative implications for factor income taxation are. We discover that endogenous fertility choices imply that the policy prescriptions for eliminating the allocative distortions of the capital levy may involve different tax rules from those obtained in models with an endogenous labor supply.

4.1 Closed economy

When gross investment can be deducted from taxable capital income, the tax base in levels is given by $rK - \dot{K} - \delta K$ (where K is the level of the capital stock). Therefore, the dynamic budget constraint of consumers, expressed in per capita terms, is given by

$$\dot{k} = (r - \delta - n)k + \frac{[(1-\tau_l)wl + q - c]}{(1-\tau_k)}. \tag{21}$$

The first-order conditions for the maximization of (1) subject to (3) and (21) are

$$\frac{U_n}{U_c} = (1 - \tau_l) w T' + (1 - \tau_k) k, \tag{22a}$$

$$-\frac{d}{dt} \ln U_c = r - \delta - n - \rho. \tag{22b}$$

In this case, factor income taxation involves only static distortions as the intertemporal tax wedge on capital income disappears. By embedding equations (22) into the general equilibrium model, it can immediately be seen that the first-best allocation is replicated if $\tau_k > 0$ and $\tau_l = -\frac{k}{wT'}\tau_k < 0$.[21] This result, which is the exact opposite of our closed economy one, also differs from the Abel (2006) finding, which instead implies that labor income should be tax free.

This discovery can be summarized as follows:

Proposition 4 *In a model with endogenous fertility, capital income taxation combined with the exemption of capital expenditure requires that labor should be subsidized with the scope of replicating the first-best allocation.*

4.2 Small open economy

As the Abel (2006) tax proposal contemplates *de facto* a full expensing of saving, i.e. gross capital accumulation, in an open economy we have to

[21] Abel (2006) shows that the Hall-Jorgenson proposal of capital income tax generates the first-best equilibrium as a proposal based on consumption and labor tax mix, if the normative prescription $\tau_l = -\tau_c < 0$ is satisfied (where τ_c is a proportional consumption tax rate); that is, consumption and leisure must be taxed at a uniform rate. It is not difficult to show that in our setup the co-existence of consumption and labor taxation implies that for a positive consumption tax, the optimal labor subsidy rate that replicates first-best allocation is given by $\tau_l = -\frac{(F_l T' + k)}{F_l T'}\tau_c < 0$. Note that this result violates the Atkinson-Stiglitz (1972) principle that all commodities (and hence fertility) should be uniformly taxed (this is because the capital stock enters the implicit cost of fertility).

consider the full expensing of wealth accumulation in order to eliminate the intertemporal wealth tax wedge. In an open economy, the distortionary role of capital taxation can be eliminated if the fiscal regime adopted is residence-based. In this case, taxable income from wealth is given by $r(K+B)-\dot{K}$ $-\delta K-\dot{B}$. The flow budget constraint of consumers can be written as

$$\dot{k}+\dot{b}=(r^{*}-n)(k+b)+\frac{[(1-\tau_{l})wl+q-c]}{(1-\tau_{a})}. \tag{23}$$

The basic equations describing the agent's optimal choices are

$$\frac{U_n}{U_c}=(1-\tau_l)wT'+(1-\tau_a)(k+b), \tag{24a}$$

$$-\frac{d}{dt}\ln U_c=r^{*}-n-\rho. \tag{24b}$$

From equations (24), it is immediately clear that also in a small open economy this type of nonhuman wealth taxation is non-distortionary if labor is subsidized at a rate $\tau_l=-\frac{(k+b)}{wT'}\tau_a<0$.

In the case of a source-based capital tax regime, the Abel-Hall-Jorgenson first-best proposal does not hold as the tax base is now given by income from capital expensed from gross saving.

The following proposition summarizes our results

Proposition 5 *In a small open economy with endogenous fertility and perfect capital mobility, optimal residence-based capital income taxation with immediate expensing of wealth accumulation implies that the labor tax rate should be negative to get nondistortionary capital taxation. A territorial capital income tax with saving expensing is not capable of reproducing the first-best equilibrium.*

5 Concluding remarks

This paper has investigated the question of optimal factor income taxation in intertemporal optimizing models of wealth formation with endogenous population growth. The analysis has considered infinitely-lived closed and small open economies.

The consideration of elastic fertility choices may invalidate the Chamley-Judd normative prescription of a zero capital income tax rate found in a neoclassical growth model with endogenous labor-leisure choices. In fact, in a closed economy as well as in a small open economy operating under a perfectly integrated capital market and adopting a regime of residence-based wealth taxation, welfare maximization implies that income from capital should be subsidized. If instead a small open economy that adopts a source-based regime of taxation is taken into account, capital income should be exempted from taxation.

Also the Abel (2006) proposal of accompanying capital taxation with the exemption of gross investment from taxable income has been investigated under the hypothesis of endogenous population growth. Our results depart, once again, from those obtained in models with an elastic labor supply. We discover that the first-best allocation can be replicated by adopting wealth income taxation (that incorporates saving exemption) coupled with labor income subsidization. This is valid for a closed economy and a small open economy adopting a system of worldwide taxation.

Our general findings are quite surprising as infinitely-lived models with endogenous fertility are similar to the corresponding models incorporating endogenous labor-leisure choices. In particular, the similarity between the two setups is due to the apparently parallel role of fertility and leisure: Fertility is at the same time a good as well as indirectly an input, like leisure. Differently from leisure, fertility enters the consumers' budget constraint not only through the time allocation constraint, but also through a nonlinear term (given by the population growth rate times the capital intensity) that reflects the reduction in the capital-labor ratio due to population growth. This element generates a demand for fertility that depends on financial wealth. Since the static efficiency condition for fertility enters the implementability constraint and hence the pseudo-welfare function of the social planner, the stock of wealth appears directly in the maximand function of the "Ramsey problem", thus altering the Chamley-Judd optimal capital income tax rule (except for the case of a small open economy adopting a territorial system of taxation).

Therefore, this theoretical contribution has identified another source of invalidity of the Chamley-Judd zero capital tax principle. This source of invalidity does not come from the supply-side — as, for example, occurs in the cases analyzed by Correia (1996a), and Jones, Manuelli and Rossi (1997) — but from the demand-side through the effects of fertility on preferences and the budget constraint of consumers.[22]

[22]Relatively to a small open economy, it can be curiously noticed that the consideration of endogenous fertility choices entails a capital tax rate different from zero in the sole case of a residence-based regime, while the introduction of an additional untaxed input in a model with an endogenous labor supply alters the result *à la* Chamley-Judd only when capital taxation follows a source-based principle (Correia, 1996b).

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