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Neoclassical Growth, Manufacturing Agglomeration and Terms of Trade

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Neoclassical Growth, Manufacturing Agglomeration, and Terms of Trade*

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Abstract

This paper presents an integrated view of economic growth, development traps, and economic geography. We explain why there is income convergence among some countries (neoclassical regime) and income divergence among others (poverty trap regime). Income convergence (divergence) and manufacturing industry diffusion (agglomeration) are re-enforcing each other in a cumulative process. Moreover, trade openness may trigger a catch-up process of an economy that is stuck in a "poverty trap". This catch-up is characterized by an increase in the investment-to-GDP ratio and an improvement of the terms of trade. A new dynamic welfare gain of trade liberalization is identified, which is likely to be large.

JEL Classification: F12, O41
Keywords: agglomeration, complementarities, convergence, dynamic trade theory, dynamic welfare gains of trade, poverty trap, terms of trade, trade liberalization

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1 Introduction

"... many people still vaguely believe that manufacturing somehow matters more than any other economic activity; ... Manufacturing, in this way of looking at things, brings more growth, better-paid jobs, fatter export earnings and greater technological progress than any other economic activity." (Carson, 1998, p. 2)

Manufacturing activity is special, because it is more likely to be characterized by increasing returns to scale technology than many other economic activities (Kaldor, 1967). However, increasing returns to scale di¤ers from constant returns to scale technology in the general equilibrium outcome. Increasing returns to scale helps explain intra-industry trade (Krugman, 1979, 1980), agglomeration of manufacturing industries (Krugman, 1991a), specialization without comparative advantage or endowment di¤erences (Krugman and Venables, 1995, 1996), sustained growth (Romer, 1986), big push (Murphy, Shleifer, and Vishny, 1989), among many others.[1]

These implications of increasing returns to scale technology are also interrelated among each other. For example, Baldwin and Forslid (1998) argue: "It is a pervasive fact that industrial agglomeration and growth are closely linked..." (p. 701) Empirically, this interrelation has been clearly demonstrated by Gallup and Sachs (1998). Theoretically, there have recently been a number of endogenous growth models which are merged with economic geography models or industrialization models. Chen and Shimomura (1998) analyze industrialization by technology adoption in the presence of learning externalities. Wong and Yip (1998) look at specialization patterns in manufacturing and agriculture and its welfare implications, if manufacturing also involves a learning externality. Local inputs with scale economies have been integrated into a two-country exogenous growth model by Faini (1984) and into an endogenous growth model by Englmann and Walz (1995). Walz (1998) considers economic integration by a new country entering a trade-bloc in a geography and growth model. Kelly

[1] An extensive account of the implications of increasing returns is given in Buchanan and Yoon (1994).

(1997) argues that trade liberalization may trigger a process of growth take-o¤ yielding agglomeration (and growth) clusters in an intermediate stage as more and more infrastructure investments bind together ever larger areas of clusters. Martin and Ottaviano (1999) show the impact of the degree of agglomeration on the growth rate of the economy in the presence of local and global information spillovers. Martin and Ottaviano (1996) explore the circular causality between R&D lab location decisions and manufacturing industry location decisions. Baldwin (1999) explores the impact of Krugman's (1980) home market e¤ect on Tobin's q and knowledge capital accumulation. Baldwin, Martin, and Ottaviano (1998) provide a synthesis of R&D spillover e¤ects and home market e¤ect that is used to explain di¤erent stages of development. These endogenous growth models are well suited to explain geographic poverty traps, trade related barriers to industrialization, and the lack of R&D and manufacturing in LDCs in a North-South world.

Instead, we would like to focus on a North-North world with similar countries to explore two types of phenomena: 1) the fall-back of a developed country relative to others, and 2) the impact of trade liberalization on the convergence or divergence of income across two countries. Occassionally, some well-developed countries appear to fall behind and this process is related to manufacturing. For example, there exists a large literature on the decline of British manufacturing up to the end of the 70ies.[2] In this case, a country may not loose its entire manufacturing sector as is the eventual outcome of most endogenous growth models, but may end up with a lower growth performance in manufacturing output than other countries.[3] Also, physical capital accumulation may play a dominant role relative to human capital and R&D.[4] Eventually, it may be more interesting to study a smooth transition path towards a new steady state in the fall-back process, rather than considering a "catastrophic jump"

[2] See, for example, Edgerton (1996), Kitson and Michie (1996), and Bean and Crafts (1995) to just mention a few.

[3] Kitson and Michie (1996) show that average annual manufacturing output grew 1.5 % in the UK in comparison to 3.9 % in the US or 2.7 % in Germany in the period from 1964 until 1989.

[4] For the British decline, this has been demonstrated in Bean and Crafts (1995) using Levine and Renelts (1992) growth regression results. A shortage of physical capital accumulation accounts for 0.55 percentage points of a total of a 1.63 % annual UK growth rate shortfall compared to a mere 0.08 percentage points explained by a lack of education (human capital). However, there is some evidence that a lack of R&D also plaid an important role (see Edgerton, 1996).

in the growth rate, which is of interest in the LDC case. All these arguments indicate that the study of a neoclassical growth and geography model is empirically useful.

The study of the impact of trade liberalization on income convergence of developed countries also requires a deviation from the existing models. Trade liberalization causes income divergence in geography and growth models[5], but empirically, income convergence has been found in the European integration process or the North-American trade liberalization process (Ben-David, 1993), whereas trade liberalization of LDCs may have quite the opposite implication (see Rauch, 1997, for some examples). Also, the convergence process triggered by trade liberalization peters out after a while (Ben-David, 1993). Hence, the transition path to the new steady states after trade liberalization is more important than the study of the change of the steady state growth rate. Again, this points at the usefulness of a neoclassical growth and geography model.

To capture the above stylized facts in a North-North world, we merge a 2-country neoclassical growth model with an economic geography model. The country that grows stronger improves its relative producer prices (terms of trade) because of the home-market e¤ect (Krugman, 1980). The savings and investment decision in each country is based on the present and future real interest rate which is equal to the real rental rate of capital. The real rental rate in each country at a given point in time is in‡uenced by three e¤ects: (i) The higher producer prices in the larger country allow ceteris paribus for higher rental rates (agglomeration force I). (ii) Because trade cost are lower in the larger country (proximity to production cluster), there is cheaper access to the consumption basket in the larger country. Hence, the consumption price index is lower in the larger country. This means - everything else equal - that the real interest rate is higher in the larger country (agglomeration force II). (iii) The capital-labor ratio is higher in the larger country. By capital-labour substitutability, this implies a higher wage-rental rate in the larger country (convergence force). The

[5] An exception is Walz (1998). However, trade integration is di¤erently de…ned by the entrance of a new country into a trading-bloc.

net effect of the three forces turns out to be ambiguous and depends on the level of trade costs.

If trade costs are sufficiently large (poverty trap regime), then the real rental rate of capital of the larger country is larger than of the smaller one which induces even more capital accumulation in the larger country relative to the smaller one (complementarity of investment projects). Income is diverging and the smaller country suffers from lower welfare. If trade costs are small (neoclassical regime), then the real rental rate of capital is larger in the smaller country and the smaller country accumulates more capital (substitutability of investment projects). Income converges among the two countries.

Our first contribution will be to focus on a new agglomeration process of manufacturing industries among countries that is based on a mutual interaction with physical capital accumulation and growth. Agglomeration of economic activity on different levels like city, region, or nation may be explained by different agglomeration forces.[6] Cities may be formed by localized intermediate inputs (Abdel-Rahman (1988), Fujita (1988), Rivera-Batiz (1988), and in a growth setting Englmann and Walz (1995)). Disparities among regions may be caused by factor movements such as worker migration (Krugman, 1991a), or forward and backward linkages caused by intermediate goods (Venables, 1996). Internationally, frictionless factor movements are less likely to happen than interregionally.[7] What causes thus an unequal distribution of manufacturing industries among countries? One answer is specialization of countries in different sectors (e.g. Krugman and Venables, 1995); another one is information externalities (Grossman and Helpman, 1991); and a third answer is R&D location decisions (Martin and Ottaviano, 1996). The simplest explanation is, however, that there are more manufacturing firms in one country relative to another, because this country has accumulated more physical capital. This alone does not suffice for an explanation. The missing part is how firm agglomeration feeds back on diverging capital accumulation.

[6] Fujita and Thisse (1996) survey the literature on agglomeration economics. We consider only endogenous explanations in cumulative processes, such that completely identical countries end up diverging from each other if there is just a small disturbance (idiosyncratic shock).

[7] See Krugman and Venables (1995).

How does an increase of agglomeration lead to higher growth of a country relative to another, and higher growth to even higher firm agglomeration? We explain this feed-back with a terms-of-trade effect. In our model, the country that grows faster improves its terms of trade. In contrast, the terms of trade remain constant in other geography and trade models. In other dynamic trade models - such as Osang and Pereira (1997) - the terms of trade deteriorate, if a country grows faster than another.

Our second contribution will be to explain how trade-liberalization triggers a catch-up process. It is obvious that the agglomeration forces depend crucially on the costs of bridging distances (e.g. transport cost, tariffs, information costs, etc.), because otherwise location does not matter. If agglomeration happens at a high level of trade costs and convergence at a low level, and manufacturing agglomeration or convergence feed back on growth, then we have established a (new) nexus between trade-liberalization and growth. This role of trade cost is in contrast to the typical economic geography models[8], but fits the stylized facts of Ben-David (1993) for developed countries.

Our third contribution will be to show that a country that is about to become stuck in a poverty trap can improve its welfare, if bilateral trade costs are reduced sufficiently. The latter constitutes a new source of dynamic welfare gains of trade liberalization beyond those demonstrated in Baldwin (1992). This source of a dynamic welfare gain of trade is likely to be large, because trade liberalization may trigger a (non-marginal) change of the entire time trajectory.

The rest of the paper is organized as follows: section 2 gives the formal model set-up; section 3 solves the model for the steady states; section 4 provides a stability analysis; section 4.1 discusses the neoclassical growth regime; section 4.2 discusses the "poverty trap" regime; section 4.3 discusses the model implications for economic geography; section 5 analyzes the welfare implications and identifies a new source of dynamic gains of trade from trade liberalization; section 6 concludes.

[8] An exception is Puga (1999). However, this model is not a growth model and the reason for this effect is not capital-labour substitutability of manufacturing technology - as in this paper - but land-labour substitutability of a perfectly competitive, constant returns to scale sector.

2 The Model Set-up

There are two consumers which differ only by their place of residence in two countries ($j = 1; 2$). A standard logarithmic intertemporal utility function[9] U_j is assumed that is defined on a consumption basket C_j:

$$U_j = \int_0^\infty e^{-\rho t} \ln C_j dt; \tag{1}$$

where ρ is the time preference rate, and t is a time index in continuous time.[10] The consumption basket C_j of a consumer j is of the Dixit-Stiglitz (1977) type and is defined on all domestic and foreign produced varieties with an elasticity of substitution denoted σ ($\sigma > 1$):

$$C_j = \left(\sum_{i \in N} \frac{c_{ij}^{\frac{\sigma-1}{\sigma}}}{n_1 + n_2} \right)^{\frac{\sigma}{\sigma-1}}; \tag{2}$$

where c_{ij} is consumer j's demand for variety i, N is the set of all domestically and foreign produced varieties, n_1 is the number of varieties in country 1, and n_2 is the number of varieties in country 2. Additionally, there is no international borrowing and lending and trade will have to be balanced.[11]

With monopolistic competition, each variety i will be produced by a different firm i. Firms differ only by their location. Therefore, firms within a country j are symmetric and the index i for firm i in country j can be collapsed to j denoting a typical firm in country j:

The production technology is a Cobb-Douglas production function with fixed cost that gives rise to increasing returns to scale on plant level. In particular, α units of inputs v_j in form of a basket of labour l_j and capital k_j are used to install the

[9] All results of this paper would also hold, if an iso-elastic utility function were used. But some proofs would become substantially more complicated.

[10] We suppress the time index whenever obvious.

[11] The assumption of balanced trade has a long tradition in the trade and growth literature: e.g. Stiglitz (1970), Grossman and Helpman (1991), and Ventura (1997).

production process every day (maintenance work) and β units of the input basket are used to produce each unit of goods for the domestic and the foreign market x_j:

$$v_j = \alpha + \beta x_j \qquad \text{and} \qquad v_j = k_j^{\delta} l_j^{1-\delta}; \tag{3}$$

where δ $(0 < \delta < 1)$ denotes the income share of capital.[12]

We assume as in Baldwin, Forslid and Haaland (1995) that investment and capital are the same composite of industrial goods as consumption and consumption and investment goods are interchangeable:

$$I_j = \dot{K}_j = \left(\sum_{i \in N} \frac{\iota_{ij}^{\frac{\sigma-1}{\sigma}}}{n_1 + n_2} \right)^{\frac{\sigma}{\sigma-1}}; \tag{4}$$

where I_j is the investment aggregate used by the firms in country j to increase the capital stock K_j of country j, a dot denotes the time derivative of a variable, and ι_{ij} is demand of the firms in country j for investment goods produced by a firm i. A unit of capital, i.e. a machine, may be assembled at zero cost in different ways from time-varying product spaces, but once it is assembled it performes the same service. A larger product space does not allow for more productive capital (no Smithian growth).[13] Note that we do not allow for the usual depreciation of capital. One can think of capital as a durable composite of intermediate input goods that is permanently maintained. The maintenance cost will show up in the fixed cost parameter α of the production function.

Additionally, we assume free firm entry and exit which keeps profits at zero. Production factors are immobile.[14] For simplicity, labour supply is inelastic, equally distributed among countries, and normalized to one. Finally, there are trade costs of

[12] This particular type of the production function will guarantee both constancy of factor shares (Kaldor, 1963), and constant returns to scale on industry level (Burnside, 1996).

[13] Smithian growth, i.e. the cost reduction from larger market size and increased specialization, is discussed in Kelly (1997).

[14] We make this assumption, because we want to distinguish our agglomeration process from that of Krugman (1991a), Krugman and Venables (1995), Venables (1996), and Martin and Ottaviano (1996). These papers rely on interregional or intersectoral factor (in particular labour) movements and R&D location decisions.

the Samuelson iceberg-type, such that only a fraction τ of one produced unit of a good arrives at its foreign destination ($0 < \tau < 1$).

3 Equilibrium

The consumption maximization problem of the typical agents in country 1 and 2 may be solved in two stages. First, the demand for any variety is determined for any given time path of expenditure on consumption goods. The corresponding unit expenditure function or ideal CES price index P_j is found to be:[15]

$$P_j = \left(\frac{n_j p_j^{1-\sigma}}{n_1 + n_2} + \frac{n_k p_k^{ex(1-\sigma)}}{n_1 + n_2} \right)^{\frac{1}{1-\sigma}}; \tag{5}$$

where p_j and p_k^{ex} are the domestic producer prices and export prices of firms in country j and k charged to consumers in country j, respectively. The solution to this problem[16] allows to write the individual budget constraint in the form:

$$\dot{K}_j = I_j = \frac{r_j K_j}{P_j} + \frac{w_j}{P_j} - C_j; \tag{6}$$

where r_j and w_j denote nominal rental and wage rates. Investment expenditure equals wage income and rents minus consumption expenditure. Second, the optimal consumption expenditure is determined by maximizing utility (1) taking the individual budget constraint (6), a price vector, and the initial condition as given. We assume that private agents do not foresee the impact of their behaviour on decisions of agents in the other country. This assumption excludes strategic interaction and is in line with the monopolistic competition conjecture. The optimization yields the familiar Euler equation:[17]

$$\dot{C}_j = \left(\rho_j - \lambda\right) C_j; \tag{7}$$

[15] Note that we take here the symmetry of firms within a country into account.

[16] We obtain the following relationship (and an analogous equation for the investment aggregate I_j):

$$P_j C_j \equiv \sum_{i \in N} p_i c_{ij}:$$

[17] We follow the standard procedure as in Barro and Sala-i-Martin (1995).

where $\rho_j \equiv r_j/P_j$ denotes the real rental rate of capital. Additionally, the familiar transversality condition completes the description of the dynamical system. Note that the steady state condition of the emerging dynamical system will involve equalization of real rental rates of capital across countries.

Firms maximize profits and use a mark-up pricing rule given the imperfect competition conjecture of Dixit and Stiglitz (1977) that firms take the direct impact of their price decision on goods market demand into account, but not the indirect effects on income and the price index:[18]

$$p_j = \frac{\sigma}{\sigma - 1}\beta c(w_j; r_j) \qquad \text{and} \qquad p_j^{ex} = \frac{\sigma}{\sigma - 1}\beta c(w_j; r_j)/\tau. \tag{8}$$

It is important that prices for foreign consumers contain a transport-cost mark-up on prices for domestic consumers. Furthermore, $c(w_j; r_j)$ denotes the unit cost function which is given by the following expression:

$$c(w_j; r_j) = (1 - \delta)^{\delta - 1}\delta^{-\delta} r_j^{\delta} w_j^{1-\delta}. \tag{9}$$

Finally, the relative input demand determines after aggregation the wage-rental ratio for a given capital-labour ratio (Recall that labour endowments are normalized to one.):

$$\frac{w_j}{r_j} = \frac{1 - \delta}{\delta} K_j. \tag{10}$$

Capital letters denote aggregates (e.g. $K_j \equiv n_j k_j$ and $V_j \equiv n_j v_j$). Additionally, the zero profit condition $n_j p_j x_j = r_j K_j + w_j$ holds due to free firm entry and exit. Hence, we find from the zero profit condition and equation (10) that the rental payments are a constant fraction of income:

$$r_j K_j = \delta n_j p_j x_j. \tag{11}$$

Using the zero profit condition, we derive the following equation for firm output:[19]

$$x_j = \bar{x} = 1; \tag{12}$$

[18] For a discussion of this conjecture see d'Aspremont, et. al. (1996).

[19] For the derivation, we use the definition of V_j, equations (3) and (8), and $c(w_j; r_j) V_j = r_j K_j + w_j$ which is obtained by plugging (11) and its counterpart for labour demand into the definition of V_j, deviding through by $c(w_j; r_j)$, and applying the zero profit condition.

where we normalized without loss of generality $\beta = 1 - \alpha$ and $\alpha\sigma = 1$.[20] Factor market equilibrium requires:

$$n_j = K_j^{\delta}/V_j. \tag{13}$$

Thus, the number of firms and goods depends on the capital stock of a country. The goods market equilibrium condition for a typical firm in country 1 at any point of time is the last equilibrium condition to be imposed:[21]

$$\frac{p_1^{-\sigma}(r_1K_1 + w_1)}{n_1p_1^{1-\sigma} + qn_2p_2^{1-\sigma}} + \frac{qp_1^{-\sigma}(r_2K_2 + w_2)}{qn_1p_1^{1-\sigma} + n_2p_2^{1-\sigma}} = 1. \tag{14}$$

where $q \equiv \tau^{\sigma-1}$ proxies the reciprocal of trade costs for notational simplicity. Using the zero profit condition and defining relative producer prices (terms of trade) $p \equiv p_2/p_1$ and relative firm agglomeration $n \equiv n_2/n_1$, equation (14) can be reformulated in the following way:

$$\frac{1}{1 + qnp^{1-\sigma}} + \frac{qnp}{q + np^{1-\sigma}} = 1, \tag{15}$$

which can be solved for n to give two solutions $n = 0$ and

$$n = \frac{q - p^{\sigma}}{p(q - p^{-\sigma})} \qquad \text{with} \qquad 0 < n < \infty \tag{16}$$

This simple equation gives a relationship between the terms of trade and relative firm agglomeration.

Defining $K \equiv K_2/K_1$, equation (13) may be restated in the following way:

$$n = K^{\delta}. \tag{17}$$

The degree of firm agglomeration is determined by the relative size of capital stocks. From now on, we can use firm agglomeration n and relative capital stocks K interchangeably. Next, the relative consumption price index P (real exchange rate) of the two countries can be written after some manipulations as:

$$P = p^{\frac{\sigma}{1-\sigma}}, \tag{18}$$

[20] All results of the model are independent of α and β.

[21] Note that we exploit here the fact that the composition of consumption good and investment good demand is irrelevant for goods market equilibrium, because we assumed investment and the consumption basket to be of the same functional composite of goods.

where we used (5) and (16). Define relative (nominal rental rates) $r \equiv r_2/r_1$. Then, it follows from (11), (13) and (17) that

$$r = pK^{\delta - 1} \tag{19}$$

The relative (nominal) rental rate depends on two factors: the relative capital stocks and the relative producer terms of trade. Now, we can summarize the factor and goods market equilibrium conditions in the following Lemma.

Lemma 1: For $0 < K \leq 1$ holds: the correspondence $p = p(K)$ is an upward sloping function below 1; $P = P(K)$ is a downward sloping function above 1; $r = r(K)$ is bounded from below by $p(K)$; Finally, $\lim_{K \to 0} r(K) = \infty$.

Proof: See appendix 1. Q.E.D.

Lemma 1 can be shown in figure 1 that depicts the terms of trade $p(K)$, relative rental rates $r(K)$, and the relative consumption price index $P(K)$ in dependence of the degree of relative capital stocks K. Note additionally that relative capital stocks K and firm agglomeration n are proportional (equation (17)).

Figure 1 about here

If industries are partially agglomerated in country 1 ($K < 1$), then the terms of trade $p(K)$ are larger in country 1, whereas the consumption price index $P(K)$ is smaller. However, the relation of rental rates $r(K)$ to relative capital stocks K may be ambiguous.

These results reflect the interplay between terms of trade and agglomeration of industries that is implicit in Krugman (1991a). Suppose, the economy starts from an equal distribution of industries. Then, the relative distribution of production factors changes, because one country is accumulating more capital. Consequently, there will be more purchasing power in the larger country than in the smaller one. Because of trade costs, demand for goods of a typical firm is biased towards domestic firms. This implies that demand for goods of a typical firm in the larger country exceeds the one in the smaller country. However, supply of firms is the same across all firms in the Dixit and Stiglitz (1977) framework (see equation (12)). Thus, goods market clearing requires that relative producer prices fall in the smaller country. The price movement induces the exit of firms in the smaller country and the entry of new firms in the larger (see equation (17)).

The consumption price index of a typical consumer in the large country is below the one in the small country, although (factory gate) producer prices are higher in the large country and a larger share of income is spent on domestic goods (See equation (18)). This is so, because less goods have to be imported in the large country. Hence, there are less goods a transport-cost mark-up has to be paid for. (See equation (8)). In this sense, transport cost drive a wedge between relative (factory gate) producer prices and relative consumption price indices.

The ambiguous impact of the distribution of the capital stock on rental rates arises from a convergence force, i.e. capital-labour substitutability, and from an agglomeration force, i.e. the terms-of-trade effect due to the agglomeration of manufacturing industries (home-market effect). The rise in the capital-labour ratio will lower the rental rate relative to the wage rate in the country with more capital; the rise in industrial agglomeration rises the terms of trade in the bigger country and rises the overall factor payments in factor market equilibrium including - in particular - rental rates (see equation (19)).

We close the model by combining the goods and factor market equilibrium conditions and the conditions from firm optimization with the dynamical equations from consumer optimization. Note that the intertemporal budget constraint (6) can be reformulated to yield:

$$\dot{K}_j = \frac{n_j p_j}{P_j} - C_j = \frac{r_j K_j}{\alpha P_j} - C_j; \tag{20}$$

where equation (12) is used and the second equality sign follows from equation (11). We note from (5), (11), (12), and (13), and Lemma 1 that the real rental rate of capital in a country depends on the level of the two capital stocks in the two countries K_1 and

K_2 ($\rho_j \equiv r_j/P_j = \rho_j(K_1, K_2)$). Then the model may be summarized in the following 4-dimensional, non-linear differential equation system with the control variables C_1 and C_2; the state variables K_1 and K_2, the national budget constraints (20), and the Euler equations (7):

$$\dot{K}_1 = \frac{\rho_1(K_1, K_2)}{\delta} K_1 - C_1 \tag{21}$$

$$\dot{C}_1 = (\rho_1(K_1, K_2) - \theta)\, C_1 \tag{22}$$

$$\dot{K}_2 = \frac{\rho_2(K_1, K_2)}{\delta} K_2 - C_2 \tag{23}$$

$$\dot{C}_2 = (\rho_2(K_1, K_2) - \theta)\, C_2, \tag{24}$$

where the transversality conditions are

$$\lim_{t \to \infty} K_j(t)\, \mu_j(t) = 0 \tag{25}$$

with the co-state variables $\mu_j(t)$ for (21) and (23), and the initial conditions are

$$K_j(0) = K_{j0} \tag{26}$$

for $j = 1, 2$.

Next, the steady states are calculated. Combining (22) and (24) requires $\rho \equiv \rho_2/\rho_1 = r(K)/P(K) = 1$ in the steady state. First, we define a benchmark value q^* for the reciprocal transport cost proxy q, such that

$$q^* \equiv \frac{(2\sigma - 1)(\delta\sigma + 1 - \sigma)}{\delta\sigma - (1 - \sigma)}. \tag{27}$$

Then, we can formulate the following proposition on the equalization of real rental rates of the two countries.

Proposition 1: (i) The steady state condition $\rho(K) = 1$ has the (trivial) symmetry solution $\bar{K} = 1$, if $q > q^*$; moreover, it holds that $\frac{d\rho(1)}{dK} < 0$ in this case.

(ii) The steady state condition $\rho(K) = 1$ has the solutions $\bar{K} = \{K^*, 1/K^*, 1\}$, if $q < q^*$, where $0 < K^* < 1$; moreover, it holds that $\frac{d\rho(1)}{dK} > 0$, $\frac{d\rho(K^*)}{dK} < 0$, and $\frac{d\rho(1/K^*)}{dK} < 0$ in this case.

Proof: See appendix 2.

There are two regimes depending on the level of trade costs, and one of the two regimes has multiple equilibria. The first regime will be called **neoclassical regime**; the second regime will be called **poverty trap regime**, henceforth.

Trade costs drive a wedge between relative producer prices and consumption price indices. If this wedge widens sufficiently ($q < q^*$), the intermediate solution K^* arises (see figure 1). In this case, an increase of the capital stock in the largest country rises the real rental rate above the one in the smallest country in the neighborhood of a symmetric distribution of capital ($d\rho(1)/dK > 0$). In this sense investment projects are local complements in the poverty trap regime (spatial complementarity of investment). If the wedge between producer prices and consumption price indices is not sufficiently large ($q > q^*$), then an increase of the capital stock in the largest country leads to a lower real rental rate than in the smallest country ($d\rho(1)/dK < 0$). In this sense investment projects are global substitutes in the neoclassical regime (spatial substitutability of investment).

The steady state variables $\bar{K}_1, \bar{C}_1, \bar{K}_2, \bar{C}_2$ can be obtained as functions of $\bar{K}$.[22] However, we will not focus on their values. For future reference, we will denote the set of steady state vectors $\bar{x} \equiv (\bar{K}_1, \bar{C}_1, \bar{K}_2, \bar{C}_2)$ and the particular steady state vectors associated with $\bar{K} = 1$, $\bar{K} = K^* < 1$ and $\bar{K} = 1/K^* > 1$ by x^*, x^{**}, and x^{***}, respectively. If an equation holds for any steady state vector, we will also use the notation $\bar{x}$.

Finally, we shall point at two interesting properties of the model. First, the model relies on constant factor shares which is one of the stylised facts of growth theory (Kaldor, 1963). This is so, because free firm entry and exit drives profits to zero and income is thus devided according to relative factor prices of labour and capital just as in any neoclassical model.

[22] Bars denote steady state values of a variable. Caveat: $\bar{K}$ denotes the set of all steady state capital stocks (because there are multiple equilibria), whereas K^* denotes a certain value for one particular steady state capital stock.

Second, the aggregated industry production function $n_j x_j = K_j^{\delta} L_j^{1-\delta}$ exhibits constant returns to scale. Hence, the increasing returns to scale assumption on plant level is in line with empirical evidence on the production technology on industry level such as Burnside (1996). The reason is that any expansion of industrial demand is directed entirely towards expansion of product space in the presence of love of variety, while output of a single firm remains constant in the presence of constant price mark-ups and free firm entry or exit. If firms start produce after a rise in industry demand, they make positive profits, new firms enter, but they produce a different variety in a market of monopolistic competition. Taken together, any rise in industry demand rises proportionally the number of goods. But a rise in demand comes about by a rise in income which in turn is linear homogenous in factors by the assumptions on the production factor basket v_j (3) and the result of constant factor shares.

4 Stability Analysis

We will not follow the standard procedure of a local stability analysis as in Dockner (1985) for 4-dimensional, non-linear differential equation systems, because the Jacobian of the linnearized system cannot be signed easily. Instead, we will find a first-order approximation function for the system (21)-(24) that has (i) the same steady state values, (ii) the same Jacobian matrix at the steady state values, and (iii) the Jacobian matrix is easily signed for any single entry. Finally, we use the fact that the qualitative dynamic behaviour of the approximation system is equivalent to the original system.

We take the difference in the growth rates of the capital stocks and consumption using (21)-(24):

$$\frac{\dot{K}_2}{K_2} - \frac{\dot{K}_1}{K_1} = \frac{1}{\delta}\left(\rho_2(K_1, K_2) - \rho_1(K_1, K_2)\right) - \frac{C_2}{K_2} + \frac{C_1}{K_1} \tag{28}$$

$$\frac{\dot{C}_2}{C_2} - \frac{\dot{C}_1}{C_1} = \rho_2(K_1, K_2) - \rho_1(K_1, K_2).$$

We would like to express these equations in terms of relative capital and consumption.

For this purpose, we "guess" the following approximation function to the system (28):

$$\frac{\dot{K}}{K} = \frac{a_1}{\delta} \ln \rho(K) - a_2 \ln C + a_2 \ln K \tag{29}$$

$$\frac{\dot{C}}{C} = a_1 \ln \rho(K);$$

where we defined $C \equiv C_2/C_1$, $a_1 \equiv \rho_1$, and $a_2 \equiv \bar{C}_2/\bar{K}_2$. This approximation is entirely sufficient to describe the behaviour of the terms of trade around the steady state and to pin down the relation of all state variables (capital, income, and firm distribution) between the two countries around the steady state values.[23] However, for the approximation to be valid, we need to show that the approximation (29) is chosen such that this system has the same steady states and the same qualitative dynamic behaviour as the original system (28). The first property is easily confirmed, whereas the second is proven in Lemma 2.

Lemma 2: The Jacobian matrix of the dynamical system (29), (21), and (22) evaluated at any of the steady states has the same eigenvalues as the Jacobian matrix of the dynamical system (21)-(24).

Proof: See appendix 3. Q.E.D.

This lemma will be used for the local stability analysis that is summarized in the next proposition.

Proposition 2: Consider the dynamical system (21)-(26). Assume that the eigenvalues are distinct. Then, this system is locally asymptotically stable if either

(i) $q > q^*$ and $\bar{K} = 1$ or

(ii) $q < q^*$ and $\bar{K} = K^*$ or

(iii) $q < q^*$ and $\bar{K} = 1/K^*$.

[23] To recover the absolute values of the state variables, two more equations are necessary: e.g. the dynamical equations governing country 1. We skip them to focus on the idea of the solution method, but use them in the rigorous mathematical derivation in Lemma 2 and appendix 3.

Furthermore, there exist three corresponding two-dimensional local stable manifolds $W^s_{loc}(x^{*})$; $W^s_{loc}(x^{**})$; and $W^s_{loc}(x^{***})$. On the contrary, the dynamical system (21)-(26) has a one-dimensional local stable manifold $W^s_{loc}(x^{*})$, if

(iv) $q < q^{*}$ and $\bar{K} = 1$.

This local stable manifold is described by $K_1(t) = K_2(t)$ and $C_1(t) = C_2(t)$ for $0 \quad t \quad 1$.

Proof: See appendix 4.

Proposition 2 resembles a supercritical pitchfork bifurcation with the bifurcation parameter q and the bifurcation point $q = q^{*}$: We illustrate this in the following bifurcation diagram.

Figure 2 about here

The vertical axes shows the position of steady state equilibria in terms of the relative distribution of capital; the horizontal axes shows the level of trade costs. At a high level of trade costs (low q), there are three steady states with the symmetric one ($\bar{K} = 1$) being unstable (poverty trap regime). At a low level of trade costs (high q), there is only one stable steady state equilibrium at a symmetric distribution of capital (neoclassical regime).

The poverty trap regime emerges if and only if investment projects become locally complementary in the neighborhood of a symmetric distribution of capital and firms.[24;25] Around a symmetric distribution of capital, an increase of investment in one country relative to the other increases, rather than decreases, the relative real marginal productivity of capital in terms of the consumer price indices inducing more investment to take place in the former than in the latter country. At some degree of divergence in capital stocks and firm distribution the divergence process stops, because investment projects have become locally substitutes. A further rise of investment in

[24] This follows immediately from the proof of proposition 2 in appendix 4, equations (71)-(74).

[25] Note that investment complementarity occurs endogenously in the model, whenever trade cost are relatively low, as has been explained after proposition 1 in the text. It is not required that production factors are complements.

the booming country lowers the real marginal productivity of capital relative to the declining country. Therefore the divergence process remains incomplete and a certain asymmetric distribution of capital and firms is a stable equilibrium.

The neoclassical regime emerges on the contrary, if investment projects are globally substitutes, i.e. a relative rise in investment of one country above investment in the other lowers the real marginal product of capital in the former relative to the latter country. Therefore, only the symmetric distribution of capital can be a stable steady state. Given that there can exist multiple stable local manifolds, it is important to examine one aspect of global stability.

Proposition 3: Consider the dynamical system (21)-(26) and the case $q < q^*$: For any given combination of initial conditions $K_{10}; K_{20} \in R^+$, there exists a unique perfect foresight path for the two control variables C_1 and C_2. Furthermore, x^* is approached, if $K_{10} = K_{20}$; x^{**} is approached, if $K_{10} > K_{20}$; x^{***} is approached, if $K_{10} < K_{20}$;

Proof: See appendix 5. Q.E.D.

This proposition ensures that there exists a unique perfect-foresight path. Only one of the three steady states can be reached for any given combination of initial conditions. Therefore, this model does not exhibit expectations-driven agglomeration processes as have been found in other dynamic models with increasing returns to scale like Matsuyama (1991), Krugman (1991b), and Kaneda (1995). In particular, we do not need any additional coordination mechanism of expectations as Kaneda's (1995) assumption of "euphoric expectations" to select among multiple perfect-foresight paths.

4.1 The Neoclassical Growth Regime

In this section we discuss in detail the neoclassical regime, i.e. the case where trade costs are relatively low ($q > q^*$). Recall that there is one steady state distribution of capital $\bar{K} = 1$. We summarize our results:

Result 1: The neoclassical regime ($q > q^*$) exhibits outphasing growth and convergence of income.[26]

The dynamic adjustment path is shown in figure 3.

Figure 3 about here

The figure shows the unique stable manifold of the 4 dimensional differential equation system (21)-(26). In particular, there is a unique mapping from the state space $K_2 - K_1$ to the control variable space $C_2 - C_1$ which follows from the stable manifold theorem (see proposition 2). Even if two structurally identical countries start out with dissimilar capital stocks, i.e. one country is poor and the other is rich, there will be convergence of capital stocks and per capita income. The poorer country will grow faster than the richer country in the transition period to the steady state.

Our neoclassical growth regime differs from, e.g., a Solow or a Ramsey model (without technological progress and population growth) by a different adjustment path. Thus, countries that catch up do not follow the same path as the leading countries. History does not repeat, as is the case in the Solow and Ramsey model. Once some country is ahead, the catch-up process will change terms of trade and the real marginal product of capital. This will foster income growth of the country lacking behind beyond what is predicted by a model with two isolated Ramsey economies. In this sense, the speed of convergence is higher in our neoclassical regime than in the isolated Ramsey economies.

Furthermore, our model predicts that trade-liberalization triggers a convergence process eliminating poverty traps, if q passes the threshold q^*. This adds qualitatively a new dimension to the relation between trade openness and growth convergence. The bifurcation property of trade liberalization is in line with the finding of Ben-David (1993) who shows: 1) There is absolute convergence of income in an economy with

[26] This follows from proposition 2: the steady state is stable and the relative capital stock approaches one. However, income is a monotone, increasing function of the capital stock.

trade liberalization (EEC6[27] from 1959-1968, EEC3[28] after the mid-sixties, USA and Canada after the Kennedy Round Agreement), or with trade and factor market integration (the convergence of the US states). 2) There is no absolute convergence of economies that are not integrated (e.g. the EEC6 and the EEC3 before trade liberalization, the 25 most developed countries, or the "whole world").[29] Therefore, this evidence points to a two regime scenario with trade liberalization being the bifurcation parameter as suggested by our model.[30]

The factor price equalization (FPE) theorem of international trade may also explain income convergence after trade liberalization. However, Slaughter (1997) points out for the case of the EEC3 of Ben-David (1993): "... the post-accession per-worker income convergence was driven at least partly by post-accession per-worker capital-stock convergence. The FPE theorem is about trade changing factor prices, not factor quantities." (Slaughter, 1997, p. 198.) Our explanation combines both factor price and factor quantity convergence and our model also reflects the convergence of the capital stocks, in particular. Levine and Renelt (1992) show additionally that the impact of openness on growth stems from investment promotion, and not from productivity growth.[31]

4.2 The Poverty Trap Regime

In this section, we discuss in detail the poverty-trap regime, i.e. the case where trade costs are relatively high ($q < q^*$). Recall that there are three steady state distributions of capital, one of which is unstable. We summarize our results:

[27] This is the group of countries consisting of France, West Germany, Belgium, the Netherlands, Luxembourg, and Italy.

[28] This is the group of countries consisting of Denmark, Ireland, and UK.

[29] If there is conditional convergence among the EEC6 (Barro and Sala-i-Martin, 1992), but not absolute convergence, then factors other than capital accumulation must drive income convergence. If trade liberalization causes absolute convergence, then trade liberalization must have caused a catch-up in capital stocks. This is the transmission channel in our model.

[30] The role of trade openness as bifurcation parameter may be reversed, if different convergence forces are chosen (see section 4.3). Rauch (1997) gives the examples of Chile 1974-79 and of Italy's political unification 1861, and explains the subsequent economic slumps in an endogenous growth model.

[31] Lee (1995) provides a theory and evidence that the convergence source of trade liberalization is trade in capital goods.

Result 2: In the poverty trap regime ($q < q^{¤}$), income levels tend to diverge monotonically up to some relative ratio $Y^{¤} = K^{¤±}$, if country 2 is taken to be the smaller country.[32]

The poverty trap case is graphically exposed in …gure 4 which is drawn in line with propositions 2 and 3. The …gure shows the map of the state space (initial capital distribution) on the control variable space (consumption choices) belonging to the three local stable manifolds $W^s_{loc}(x^{¤})$; $W^s_{loc}(x^{¤¤})$; and $W^s_{loc}(x^{¤¤¤})$ which are related to the three steady-state vectors $x^{¤}$; $x^{¤¤}$, and $x^{¤¤¤}$, respectively.

Figure 4 about here

Proposition 3 ensures that, for $K_1(0) = K_2(0)$; consumption is chosen in line with the stable manifold $W^s_{loc}(x^{¤})$ that leads to the symmetric steady state $x^{¤}$; if $K_1(0) > K_2(0)$; consumption is chosen in line with the stable manifold $W^s_{loc}(x^{¤¤})$ that leads to the steady state $x^{¤¤}$ with more capital in country 1; if $K_1(0) < K_2(0)$; consumption is chosen in line with the stable manifold $W^s_{loc}(x^{¤¤¤})$ that leads to the steady state $x^{¤¤¤}$ with more capital in country 2. Because $W^s_{loc}(x^{¤})$ is one-dimensional, any slight disturbance of this symmetric growth path, in the sense that one country accumulates more capital at some time period (idiosyncratic shock), will leave the symmetric steady state unachievable. Capital stocks and income will diverge governed by one of the other two stable manifolds depending on which country received a positive or negative idiosyncratic shock.[33]

The recent empirical convergence literature is inconclusive of the (conditional) convergence hypothesis (Barro, 1991, Barro and Sala-i-Martin, 1992, and Mankiw, Romer, and Weil, 1992) or the club convergence hypothesis (Baumol, et. al., 1989, Durlauf and Johnson, 1995, and Quah, 1996) for both country and regional data sets. On the one hand, Barro (1991), Barro and Sala-i-Martin (1992), and Mankiw, Romer

[32] The statement follows from proposition 2 that shows the divergence of the capital stocks and from the fact that national income is a monotonous function of capital.

[33] We cannot accomplish a global dynamic analysis, but numerical simulations suggest that a typical divergence path would stay close to the symmetric growth path for a long time after an idiosyncratic shock has occured and will eventually lead to a drastic relative and absolute decline in the country that was originally hit.

and Weil (1992) find that the average country or region converges conditionally on structural characteristics of the economies. On the other hand, Quah (1996) notes that the population of the converging regions/countries might be double peaked, thus supporting the club convergence hypothesis which says that initial conditions also matter. Additionally, Durlauf and Johnson (1995) reject the conditional convergence hypothesis in favour of multiple regimes or stages of development in a cross section analysis. Quah (1996) notes also that conditional convergence of the average country in a regression analysis is compatible with outlier countries that do not converge.[34]

A wide range of poverty trap models is at hand[35] to explain the prevalence of any sort of income divergence among some countries. Our model can be distinguished from most of the poverty trap models in a growth setting by explaining income divergence of two countries even though initial conditions are the same except for an idiosyncratic shock. In other words, the ratio of initial conditions matters, not the initial conditions themselves.

This has two implications. First, poverty trap models where absolute values of initial conditions matter[36] have difficulties explaining how the rich countries left the poverty trap, whereas the poor countries did not, if all countries started from roughly the same income levels, say in the 17th/18th century.[37] Our model allows some countries to become rich, and others, that are hit by some negative idiosyncratic shock, stay poor.

Second, our model is especially suited for explaining the fall-back of highly developed countries like the United Kingdom after the turn of the century relative to countries that had initially the same state of development. Our model catches the following stylized facts: 1) "The weakness of manufacturing industry is certainly

[34] The classical example is the Italian Mezzogiorno - a region of relative and absolute decline over decades. See Rauch (1997).

[35] A survey of these poverty trap models can be found in Azariadis (1996).

[36] These are the poverty trap models corresponding to the club convergence hypothesis. A definition and an overview of convergence hypotheses is given by Galor (1996).

[37] "The very fact that the world at present is so sharply divided between 'rich' and 'poor' countries is, in the context of the broad sweep of history, something relatively new: it is the cumulative result of the historical experience of two or three hundred years. If we go back a few hundred years for example, to 1700 or 1750, we do not find, as far as we can tell, such large differences in real income per capita between different countries or regions." Kaldor (1967, p.3)

the main reason why the UK has become a relatively poor country and why per capita incomes in the UK are now the lowest in Northern Europe." (Rowthorn and Wells, 1987, p. 224.) Kitson and Michie (1996) report that average manufacturing output in the UK grew 1.5 % from 1964 until 1989 compared to 3.9 % in the US and 2.7 % in Germany. 2) A dominant source of this weakness is the lack of capital formation. Average capital stock growth of the UK is 2.9 % compared to 3.9 % in Germany and 4 % in the US during the period from 1963 until 1982 (see Dollar and Wol¤, 1993). Bean and Crafts (1995) demonstrate on the basis of Levine and Renelts (1992) growth regression results that the shortage of UK capital formation accounts for 0.55 percentage points of a total of 1.63 percentage points growth shortfall, whereas the lack of education accounts for only 0.08 percentage points.[38]

There is still one observation to be made concerning the terms of trade which distinguishes our model from the other geography and trade models listed in the introduction and other dynamic trade models such as Osang and Pereira (1997).

Result 3: In the poverty trap regime ($q < q^{¤}$), there is a worsening of the terms of trade $p(t)$ over time in the country that lags behind vis a vis the country that is ahead, where terms of trade are de…ned in fob-manufacturing-producer prices.[39]

There has been an extensive discussion in the 50ies, whether developing countries faced a persistent worsening of their terms of trade.[40] Although - strictly speaking - our model is only suitable for developing countries whose export goods are produced with increasing returns to scale and monopolistic competition[41], our model suggests that a worsening of the terms of trade was in principle explicable, whenever investment projects were locally complements and capital accumulation was poor.[42] Our

[38] However, there is also evidence by Edgerton (1996) for a lack of R&D.

[39] Suppose country 2 lags behind. From proposition 2 follows that the relative capital stock $K(t)$ approaches assymptotically $K^{¤} < 1$. From numerical simulations can be inferred that $K(t)$ changes monotonically. From Lemma 1 follows that $p(t)$ is monotonically increasing with $K(t)$. Therefore, the time path for $p(t)$ has the same qualitative properties as the time path for $K(t)$:

[40] An empirical survey is Spraos (1980).

[41] Spraos (1980) indicates: "Perhaps more important than any of these is the processing of primary products before shipment (for instance, cocoa beans turned into cocoa butter and cocoa paste) which has been increasing all the time, though in developing countries it had gained great momentum only in the last twenty years." (p. 118) Additionally, mining and agro-business may not a priori be less likely described by increasing returns to scale than manufacturing industries.

[42] Of course, we do not doubt that other explanations can be found. We just want to point out that

model suggests that the appropriate policy meassure was not to close national markets (import substitution) despite that trade seemed to harm developing countries, but to open national markets in order to eliminate the underlying poverty trap - a recommendation that finds broad consensus nowadays.

4.3 Economic Geography

Having shown the interdependence between real marginal product of capital, capital accumulation, and terms of trade, we focus now on the aspect of agglomeration of manufacturing industries. From the analysis so far, it follows immediately (by equation (17)) that the faster growth in the country with more capital causes a larger number of firms which we take as a proxy for manufacturing industry agglomeration. A relative increase in domestic capital increases domestic income, which in turn increases demand for any existing domestic variety. The latter increases domestic producer prices relative to foreign (terms-of-trade effect), which leads to positive profits of domestic firms and thus the entry of new domestic firms.

Result 4: At high trade costs ($q < q^{*}$), there will be partial agglomeration of manufacturing industries in one country.[43]

Hence, a low growth rate is associated with a decline of manufacturing industries. Indeed, slower growth and a decline of manufacturing industries self-enforce each other in a cumulative process.[44] Conversely, agglomeration of manufacturing industries is explained by faster capital accumulation in one country relative to another. This explanation differs from other explanations in papers on agglomeration and growth - as Bertola (1992), Englmann and Walz (1995), and Martin and Ottaviano (1996). In these papers, agglomeration processes in growth models rely on migration, capital flows with technological spill-overs, and R&D location decisions.

The role of trade costs for triggering agglomeration is reversed compared to

the terms of trade effect in our poverty trap regime does not run counter to the empirical literature.

[43] This follows from result 2 and from equation (17).

[44] The notion of a cumulative process was introduced to economics by Myrdal (1957).

Krugman (1991a). This is so, because we exchanged the convergence forces. Krugman's (1991a) convergence force is based on ambiguous terms-of-trade effects caused by an immobile farming sector. As trade costs increase from a very low level, terms of trade increase in the larger country. (We observed the same effect in our model.) However, as trade costs increase further in Krugman (1991a), terms of trade may start to decrease. This effect is not present in our model. Instead, increasing trade costincrease the wedge between relative producer prices and relative consumption price indices which in turn icreases the possibility of a poverty trap. We conclude therefore that the role of trade costs is not robust with respect to the specific convergence force used in geography and trade models. A similar result to ours is obtained by Puga (1999) in an economic geography model. The latter model assumes land-labour substitutability of the constant returns to scale good instead of capital-labour substitutability in manufacturing.

5 Dynamic Welfare Gains of Trade Liberalization

This section asks the following questions on the welfare implications of the model: Does the country that is about to become stuck in a poverty trap have a lower intertemporal welfare than the country that is forging ahead? Suppose a country is about to become stuck in a poverty trap and income is diverging. Will there be a welfare gain of trade liberalization (i.e., a reduction in trade barriers), if the regime switches from the poverty trap to the neoclassical one, and income divergence is turned into convergence? Will this welfare gain go beyond the one known from Baldwin (1992)?

We first explore, whether the "large" country is better off in the poverty trap regime ($q < q^*$) approaching the large-country steady state than the "small" country approaching the small-country steady state.[45] Without loss of generality, let country 2 be the slightly smaller country (i.e., $K_{10} = K_{20} + \varepsilon$; $\varepsilon > 0$; $\varepsilon \to 0$). Hence, the considered steady state will be x^{**} in figure 4. We define $U_j^{**}(q)$ as the intertemporal

[45] A country is defined to be "larger" than another, if its initial capital stock (per capita) is larger.

welfare level of country j following the optimal trajectory from the intitial condition towards steady state x^{**} at a given trade cost q; i.e.:

$$U_j^{**}(q) \equiv \int_0^{\infty} e^{-\rho t} \ln C_j^{**}(t;q)\, dt; \qquad j = 1; 2; \tag{30}$$

where $C_j^{**}(t;q)$ is the period t consumption level of country j on the optimal trajectory towards steady state x^{**} at a given q. Then, we find the following proposition:

Proposition 4: In the poverty trap regime, the country that is about to become stuck in the poverty trap (country 2) will experience a lower intertemporal welfare than the country that is forging ahead. Formally, $U_2^{**}(q) < U_1^{**}(q)$; if, $K_{10} = K_{20} + \varepsilon$; $\varepsilon > 0$; $\varepsilon \to 0$; and a q is given with $q < q^*$.

Proof: See appendix 6. Q.E.D.

The "small" country does not only have less accumulation of capital, less manufacturing firms, and a smaller variety of domestically produced goods, but it also has a lower welfare level than the "large" country. This result does not only hold for the steady state, but also for the transition path of two countries with (almost) the same initial conditions and the same structural characteristics. Thus, a very small shock may cause a large intertemporal welfare gain or loss.

The intuition for this result is straight forward: the country that approaches the large-country steady state experiences a persistent improvement of its terms of trade, whereas the country that approaches the small-country steady state experiences a deterioration. Hence, the large country can afford to consume more goods over time, whereas the small country can afford to consume less (direct effect). Additionally, the large country has a larger real return on investment in the face of this terms of trade trend and chooses to accumulate more capital than the small country. This accumulation effect also increases the feasible budget set from which the large country makes its consumption and investment choice (indirect effect).

Second, we address the question whether a switch from a diverging growth path to a symmetric growth path improves welfare in the neoclassical regime, if 2 countries start from (almost) the same initial condition. We define $U_2^{*}(q)$ as the welfare level of country 2 at a given trade cost q according to equation (1), if consumption follows an optimal trajectory from some initial condition $K_{10} = K_{20}$ towards the symmetric steady state x^{*} that is depicted in figures 3 and 4. Hence, $U_2^{*}(q)$ is the maximal welfare of country 2, if the two countries grow identical. Then, the proposition holds:

Proposition 5: Suppose two countries face the same initial capital stocks (up to some infinitesimally small idiosyncratic shock) in the poverty trap regime. Then, the country on the diverging growth path towards a small-country steady state (country 2) can increase its welfare, if it manages to switch to a symmetric growth path with identical steady states for both countries. Formally, $U_2^{**}(q) < U_2^{*}(q)$; if $K_{10} \approx K_{20}$; and a q is given with $q < q^{*}$.

Proof: See appendix 7. Q.E.D.

The reason is again the different time path of the terms of trade in the two cases. Whereas there is no change in the terms of trade on the symmetric growth path towards the steady state x^{*}, there is a deterioration of the terms of trade of the country on the diverging growth path towards the small-country steady state x^{**}. Again, the direct effect and the indirect effect apply: 1) the consumption basket becomes ever more expensive on the diverging growth path and less consumption is affordable. 2) Capital accumulation is smaller on the diverging growth path and the period budget constraint grows slower over time.

Finally, we consider the following thought experiment: Suppose two countries are (almost) identical, but country 2 is expecting to be on a diverging growth path towards the small-country steady state x^{**}. What is its welfare gain of a reduction of trade cost, say from q to q', if this reduction is sufficient to switch the regime from the poverty trap to the neoclassical one ($q < q^{*} < q'$)? The resulting welfare gain can be split into two parts: first, the welfare gain from a divergent growth path to a symmetric one at the same trade cost level ($U_2^{*}(q) - U_2^{**}(q)$); second, the welfare gain from a reduction of trade cost on a symmetric growth path $(U_2^*(q') - U_2^*(q))$. The first source of a welfare gain is dealt with in proposition 5. The second source of a welfare gain is explored in Baldwin (1992). According to the latter study, there is a static welfare gain due to cheaper access to foreign goods. There is also a dynamic welfare gain, because trade liberalization increases capital accumulation which is too low in the presence of monopoly pricing.

Thus the welfare gain due to a jump out of a poverty trap (as in proposition 5) is a new source of a dynamic welfare gain of trade liberalization additionally to Baldwin (1992). This dynamic welfare gain is likely to be large, because it is not just caused by a marginal re-allocation of factors, but by choosing a completely different time trajectory. Such a dynamic welfare gain will be present, whenever there is a structural break in an investment time series after trade liberalization.

6 Conclusion

We built a model that explains income divergence in a poverty trap regime, income convergence in a neoclassical regime, and a testable condition under which a country is in one or the other regime. This condition depends on the degree of integration in product markets. If trade barriers are high, income divergence is likely to occur. If trade barriers are low, income convergence is the unique equilibrium. Thus, trade liberalization may trigger a catch-up process of countries that are stuck in a poverty trap.

The interrelation of growth and agglomeration is described by circular causation. Countries grow faster (slower), because they have a lot of (a few) manufacturing industries. Countries have a lot of (a few) manufacturing industries, because they have grown faster (slower) in the past and thereby accumulated more (less) capital. The circular causation relies on a terms-of-trade effect that may or may not feed back on real rental rates (spatial complementarity or substitutability of investment). The countries stuck in the poverty trap experience slower growth, a lower investment-to-GDP ratio, a worsening of their terms of trade, and a decline in manufacturing industries. The countries that catch up experience a higher growth rate, a higher investment-to-GDP ratio, an improvement of their terms of trade, and manufacturing industries diffuse to the poor country.

The country that is about to become stuck in a poverty trap experiences a lower welfare than the country that is forging ahead. A country that is about to become stuck in a poverty trap can gain by trade liberalization, since this may turn the process of income divergence into a process of income convergence. Such a dynamic welfare gain of trade liberalization due to a jump out of a poverty trap is additional to the welfare gains explored in Baldwin (1992).

The results in this paper have been derived in a specific model set-up - increasing returns and monopolistic competition. Following the same sort of argument as Gali (1994) for a closed economy, the pitchfork-bifurcation property may also appear in a set-up with Cournot oligopoly. Our analysis suggests that divergence of income and firm agglomeration emerge, whenever investment projects are complementary in the neighborhood of a symmetric distribution of capital and firms.

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Appendix

Appendix 1: Proof of Lemma 1.

Taking the derivative of (16) yields:

$$\frac{dn}{dp} = \frac{-\sigma p^{\sigma}(q - p^{-\sigma}) - (q - p^{\sigma})(q - (1-\sigma)p^{-\sigma})}{p^2 (q - p^{-\sigma})^2} \tag{31}$$

with $0 < n < \infty$: (a) Suppose $p > 1$, then $q - p^{\sigma} < 0$. Therefore, $q - p^{-\sigma} < 0$ for n to be positive by inspection of (16). Then follows by inspection of (31) that $dn/dp > 0$, because $\sigma > 1$ by assumption. (b) Suppose $p < 1$, then $q - p^{-\sigma} < 0$. Therefore, $q - p^{\sigma} < 0$ for n to be positive by inspection of (16). Then follows by inspection of (31) that $dn/dp > 0$, because $\sigma > 1$ by assumption. This implies that the invers function exists in the positive range. From (17) follows that K and n are proportional. Hence, we have that $p(K)$ is an invertible function and $dp/dK > 0$. $P(K)$ must then be downward sloping from (18). From (19) follows that $r(K)$ is bounded from below by $p(K)$. Finally, the limit with respect to complete agglomeration ($K \to 0$) can be taken from (19). Q.E.D.

Appendix 2: Proof of Proposition 1.

We will first show that there are at most 3 solutions to $\rho(K) \equiv r(K)/P(K) = 1$. Using equations (18) and (19) yields:

$$r/P = p n^{\frac{\delta-1}{\delta}} p^{\frac{\sigma}{1-\sigma}} = 1. \tag{32}$$

Plugging in the goods market equilibrium condition (16) yields:

$$p^{\frac{1}{1-\sigma}} \left(\frac{q - p^{\sigma}}{p(q - p^{-\sigma})} \right)^{\frac{\delta-1}{\delta}} = 1. \tag{33}$$

Multiplying out gives a power function of the form:

$$q - p^{\sigma} - q p^{\frac{1-\sigma+\sigma\delta}{(1-\sigma)(1-\delta)}} + p^{\frac{(1-\sigma)^2(1-\delta)+\delta}{(1-\sigma)(1-\delta)}} = 0. \tag{34}$$

This expression has at most 3 solutions for p due to Descartes' Rule of Sign. Because there is a one-to-one mapping from p to n to K (Lemma 1), there correspond at most three values for n and K. We conclude: one solution is $K = 1$ (The symmetry solution is always true.); if there is a second solution $K^* < 1$, then the third must be $(1/K^*) > 1$ because of the symmetry of the model.

Now, we will give a necessary and sufficient condition for the existence of K^* by restricting our view on $0 < K \leq 1$. Recall from Lemma 1: $\lim_{K \to 0} r(K) = \infty$, whereas $P(0)$ is finite. Hence,

$$\lim_{K \to 0} \rho(K) \equiv \lim_{K \to 0} \frac{r(K)}{P(K)} > 1. \tag{35}$$

There will exist the interior solution K^*, if $\rho(K) < 1$ for K slightly below 1 (intermediate value theorem). This is not just a necessary condition for the existence of K^*, but also a sufficient condition for K^* to be the only interior solution ($0 < K^* < 1$), because $\rho(1) = 1$. (Suppose on the contrary that K^* exists and $\rho(K) > 1$; when K is slightly below 1, then there will exist at least two interior solutions (or none) for $0 < K < 1$ which contradicts our findings above.) From (17) follows that there corresponds a $n^* = K^{*\delta}$. We can formulate the necessary and sufficient condition for an interior solution n^* also in the following way:

$$\frac{dP(1)}{dn} < \frac{dr(1)}{dn}. \tag{36}$$

Evaluating the derivative of the relative price index yields:

$$\frac{dP(n)}{dn} = \frac{\sigma}{1-\sigma}\frac{dp(n)}{dn}. \tag{37}$$

Evaluating this expression at $n = 1$ and using equation (31) gives us:

$$\frac{dP(1)}{dn} = \frac{\sigma}{1-\sigma}\frac{q-1}{1-2\sigma-q}. \tag{38}$$

Next, the derivative of relative rental rates is found:

$$\frac{dr(n)}{dn} = \frac{dp(n)}{dn} n^{\frac{\delta-1}{\delta}} + p\left(\frac{\delta-1}{\delta}\right) n^{-\frac{1}{\delta}}. \tag{39}$$

We evaluate this expression at $n = 1$ by using (31):

$$\frac{dr(1)}{dn} = \frac{q-1}{1-2\sigma-q} + \frac{\delta-1}{\delta}. \tag{40}$$

Using (38) and (40) in (36) yields an inequality

$$\frac{q-1}{1-q-2\sigma}\frac{2\sigma-1}{1-\sigma} < \frac{\delta-1}{\delta}, \tag{41}$$

which can be solved for q:

$$q < q^* \equiv \frac{(2\sigma-1)(\delta\sigma+1-\sigma)}{\delta\sigma-(1-\sigma)} < 1. \tag{42}$$

A similar argumentation holds for $1/n^*$ and $1 \le n < \infty$, i.e. $1/K^*$ and $1 \le K < \infty$, by the symmetry property of the model.

Finally, the derivative in (i) follows from $\rho(1) = 1$; $\rho(K) > 1$, if $K < 1$; and $\rho(K) < 1$, if $K > 1$. Correspondingly, the derivatives in (ii) follow from $\rho(1) = \rho(K^*) = 1$; and $\rho(K) > 1$, if $K < K^*$ or $K > 1/K^*$; $\rho(K) < 1$, if $1 > K > K^*$ or $1/K^* > K > 1$. (See Lemma 1). Q.E.D.

Appendix 3: Proof of Lemma 2.

Let $x \equiv (K_1, C_1, K_2, C_2)$ and the dynamical system (21)-(24) be written in matrix notation as $\dot{x} = f(x)$. Furthermore, let $y \equiv (\ln C, \ln K, K_1, C_1)$ and

$$g(y) \equiv \begin{pmatrix} a_1 \ln\rho(K) \\ \frac{a_1}{\delta}\ln\rho(K) - a_2 \ln C + a_2 \ln K \\ \frac{\tilde{\rho}_1(K_1,K)}{\delta} K_1 - C_1 \\ \tilde{\rho}_1(K_1,K) - \lambda C_1 \end{pmatrix}; \tag{43}$$

where we use

$$\rho_1(K_1,K_2)=\hat{\tilde{\rho}}_1(K_1,K)\equiv\delta K_1^{\delta-1}\left(\frac{1}{1+K^{\delta}}+\frac{qp(K)^{1-\sigma}}{1+K^{-\delta}}\right)^{\frac{1}{\sigma-1}},\tag{44}$$

with $\partial\hat{\tilde{\rho}}_1/\partial K_1<0$, which follows from (5), (11), (12), and (13).[46] Define the invertible matrix h in the following way:

$$h\equiv\begin{pmatrix}0 & -\frac{1}{\bar{C}_1} & 0 & \frac{1}{\bar{C}_2}\\ -\frac{1}{\bar{K}_1} & 0 & \frac{1}{\bar{K}_2} & 0\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}\tag{45}$$

Then, we find for x and y in the neighborhood of $\bar{x}$ and $\bar{y}$ that

$$hx=\begin{pmatrix}\frac{C_2-\bar{C}_2}{\bar{C}_2}-\frac{C_1-\bar{C}_1}{\bar{C}_1}\\ \frac{K_2-\bar{K}_2}{\bar{K}_2}-\frac{K_1-\bar{K}_1}{\bar{K}_1}\\ K_1\\ C_1\end{pmatrix}\approx\begin{pmatrix}\ln C\\ \ln K\\ K_1\\ C_1\end{pmatrix}\equiv y,\tag{46}$$

where we used the first-order Taylor expansions

$$\ln K_2-\ln K_1\approx\frac{1}{\bar{K}_2}\left(K_2-\bar{K}_2\right)-\frac{1}{\bar{K}_1}\left(K_1-\bar{K}_1\right)\tag{47}$$

$$\ln C_2-\ln C_1\approx\frac{1}{\bar{C}_2}\left(C_2-\bar{C}_2\right)-\frac{1}{\bar{C}_1}\left(C_1-\bar{C}_1\right)$$

around the steady state vector $\bar{x}$. Furthermore, we calculate the Jacobian Matrix $B\equiv\frac{dg(\bar{y})}{dx}$ evaluated at the steady state vector $\bar{y}$.

$$\frac{dg\left(\bar{y}\right)}{dx}=\begin{pmatrix}\left(\frac{\partial\tilde{\rho}_2}{\partial K_1}-\frac{\partial\tilde{\rho}_1}{\partial K_1}\right) & 0 & \left(\frac{\partial\tilde{\rho}_2}{\partial K_2}-\frac{\partial\tilde{\rho}_1}{\partial K_2}\right) & 0\\ \frac{1}{\delta}\left(\frac{\partial\tilde{\rho}_2}{\partial K_1}-\frac{\partial\tilde{\rho}_1}{\partial K_1}\right)-\frac{\bar{C}_1}{\bar{K}_1^2} & \frac{1}{\bar{K}_1} & \frac{1}{\delta}\left(\frac{\partial\tilde{\rho}_2}{\partial K_2}-\frac{\partial\tilde{\rho}_1}{\partial K_2}\right)-\frac{\bar{C}_2}{\bar{K}_2^2} & -\frac{1}{\bar{K}_2}\\ \frac{\partial\tilde{\rho}_1}{\partial K_1}\frac{\bar{K}_1}{\delta}+\frac{\tilde{\rho}_1}{\delta} & -1 & \frac{\partial\tilde{\rho}_1}{\partial K_2}\frac{\bar{K}_1}{\delta} & 0\\ \bar{C}_1\frac{\partial\tilde{\rho}_1}{\partial K_1} & 0 & \bar{C}_1\frac{\partial\tilde{\rho}_1}{\partial K_2} & 0\end{pmatrix},\tag{48}$$

where we used the steady state conditions of (28), i.e.

$$\tilde{\rho}_1=\tilde{\rho}_2\tag{49}$$

$$\frac{\bar{C}_1}{\bar{K}_1}=\frac{\bar{C}_2}{\bar{K}_2},\tag{50}$$

and (for $j=1,2$)

$$\frac{\partial\ln\tilde{\rho}}{\partial K_j}=\frac{1}{\tilde{\rho}_2}\frac{\partial\tilde{\rho}_2}{\partial K_j}-\frac{1}{\tilde{\rho}_1}\frac{\partial\tilde{\rho}_1}{\partial K_j}\tag{51}$$

$$\frac{d\hat{\tilde{\rho}}_1}{dK_j}=\frac{\partial\tilde{\rho}_1}{\partial K_j}\tag{52}$$

[46] Recall that bars denote steady state values of any steady state solution.

$$\frac{\partial \ln \bar{K}}{\partial K_1} = -\frac{1}{\bar{K}_1} \tag{53}$$

$$\frac{\partial \ln \bar{K}}{\partial K_2} = -\frac{1}{\bar{K}_2} \tag{54}$$

$$\frac{\partial \ln \bar{C}}{\partial C_1} = -\frac{1}{\bar{C}_1} \tag{55}$$

$$\frac{\partial \ln \bar{C}}{\partial C_2} = \frac{1}{\bar{C}_2}. \tag{56}$$

Correspondingly, the Jacobian $A \equiv \frac{df(\bar{x})}{dx}$ can be found from the linnearization of the dynamical system (21)-(24) around the steady state vectors $\bar{x}$:

$$\frac{df\left(\bar{x}\right)}{dx} = \begin{pmatrix} \frac{\bar{K}_1}{\delta}\frac{\partial \rho_1}{\partial K_1} + \frac{\rho_1}{\delta} & -1 & \frac{\bar{K}_1}{\delta}\frac{\partial \rho_1}{\partial K_2} & 0 \\ \bar{C}_1 \frac{\partial \rho_1}{\partial K_1} & 0 & \bar{C}_1 \frac{\partial \rho_1}{\partial K_2} & 0 \\ \frac{\partial \rho_2}{\partial K_1}\frac{\bar{K}_2}{\delta} & 0 & \frac{\bar{K}_2}{\delta}\frac{\partial \rho_2}{\partial K_2} + \frac{\rho_2}{\delta} & -1 \\ \bar{C}_2 \frac{\partial \rho_2}{\partial K_1} & 0 & \bar{C}_2 \frac{\partial \rho_2}{\partial K_2} & 0 \end{pmatrix} \tag{57}$$

It can be checked that

$$\frac{dg\left(\bar{y}\right)}{dx} = h\frac{df\left(\bar{x}\right)}{dx}; \tag{58}$$

where we used (50) and

$$\frac{\bar{C}_j}{\bar{K}_j} = \frac{\rho_i}{\delta} \tag{59}$$

for $j = 1, 2$, which follows from the steady state conditions of (21) and (23). Now, we show that the Jacobian $A \equiv \frac{df(\bar{x})}{dx}$ and the Jacobian $B \equiv \frac{dg(\bar{y})}{dy}$ are similar matrices.[47] Therefore, we need one more preliminary calculation. From the chain rule of matrix differentiation and (46) follows:

$$\frac{dg\left(\bar{y}\right)}{dx} = \frac{dg\left(h\bar{x}\right)}{dx} = \frac{dg\left(\bar{y}\right)}{dy}h. \tag{60}$$

Because h is invertible, we can write:

$$\frac{dg\left(\bar{y}\right)}{dy} = \frac{dg\left(\bar{y}\right)}{dx}h^{-1}. \tag{61}$$

Then, we may rewrite the Jacobian B in the following way:

$$B \equiv \frac{dg\left(\bar{y}\right)}{dy} = \frac{dg\left(\bar{y}\right)}{dx}h^{-1} = h\frac{df\left(\bar{x}\right)}{dx}h^{-1} \equiv hAh^{-1}; \tag{62}$$

where the first equality sign follows from (61), the second equality sign follows from (58), and the second identity follows from the definition of A. Therefore, the matrices

[47] Definition: If A and B are square matrices, we say that B is similar to A, if there is an invertible matrix h such that $B = hAh^{-1}$. (Brock and Malliaris, 1989, p.349)

A and B are similar. However, two similar square matrices A and B have the same characteristic polynomials and eigenvalues (Theorem 4.1 in Brock and Malliaris, 1989, p.349), i.e.

$$|A - \lambda I_4| = |B - \lambda I_4| = 0. \tag{63}$$

where λ denotes the eigenvector and I_4 the 4×4 identity matrix. This concludes the proof. Q.E.D.

Appendix 4: Proof of Proposition 2.

Consider the Taylor linnearization of the system (29), (21), and (22), i.e.

$$\dot{y} = B\left(y - \bar{y}\right); \tag{64}$$

where $y \equiv (\ln C; \ln K; K_1; C_1)$, $B \equiv \frac{dg(\bar{y})}{dy}$ and $g(y)$ is defined in equation (43) of appendix 3. The characteristic polynomial of the matrix B reads:

$$|B - \lambda I_4| = \begin{vmatrix} -\lambda & a_1\frac{d\rho}{dK}\bar{K} & 0 & 0 \\ -a_2 & \frac{a_1}{\delta}\frac{d\rho}{dK}\bar{K} + a_2 - \lambda & 0 & 0 \\ 0 & \frac{\partial\rho_1}{\partial K}\frac{\bar{K}_1}{\delta} & \frac{\frac{\partial\rho_1}{\partial K}\bar{K}_1 + \rho_1}{\delta} - \lambda & -1 \\ 0 & \bar{C}_1\frac{\partial\rho_1}{\partial K} & \bar{C}_1\frac{\partial\rho_1}{\partial K_1} & -\lambda \end{vmatrix} = 0; \tag{65}$$

where λ denotes the eigenvector and I_4 the 4×4 identity matrix. Next, a Gauss-transformation with the Pivot-elements (1,1) and (3,4) is undertaken and the second column is changed with the first to form a matrix in Gauss-form.

$$\begin{vmatrix} a_3 & 0 & 0 & 0 \\ \frac{a_1}{\delta}\frac{d\rho}{dK}\bar{K} + a_2 - \lambda & -a_2 & 0 & 0 \\ 0 & a_4 & a_5 & 0 \\ 0 & \bar{C}_1\frac{\partial\rho_1}{\partial K} & \bar{C}_1\frac{\partial\rho_1}{\partial K_1} & -\lambda \end{vmatrix} = 0 \tag{66}$$

where we defined

$$a_3 \equiv a_1\frac{d\rho}{dK}\bar{K} - \frac{\lambda}{a_2}\left(\frac{a_1}{\delta}\frac{d\rho}{dK}\bar{K} + a_2 - \lambda\right)$$

$$a_4 \equiv -\frac{\bar{C}_1}{\lambda}\frac{\partial\rho_1}{\partial K} + \frac{\partial\rho_1}{\partial K}\frac{\bar{K}_1}{\delta}$$

$$a_5 \equiv \left(\frac{\frac{\partial\rho_1}{\partial K}\bar{K}_1 + \rho_1}{\delta} - \lambda\right) - \frac{\bar{C}_1}{\lambda}\frac{\partial\rho_1}{\partial K_1}.$$

Because the determinant of a matrix in Gauss form is the product of its diagonal elements, the characteristic polynomial may be written in the following way:

$$\left[\lambda^2 - \lambda\left(\frac{a_1}{\delta}\frac{d\rho}{dK}\bar{K} + a_2\right) + a_1a_2\frac{d\rho}{dK}\bar{K}\right]\left[\lambda^2 - \frac{\lambda}{\delta}\left(\frac{\partial\rho_1}{\partial K}\bar{K}_1 + \rho_1\right) - \bar{C}_1\frac{\partial\rho_1}{\partial K_1}\right] = 0 \tag{67}$$

Correspondingly, the 4 eigenvalues are:

$$\lambda_{1;2} = 0.5\left(\frac{a_1}{\delta}\frac{d\rho}{dK}\bar{K} + a_2\right) \pm 0.5\left(\left(\frac{a_1}{\delta}\frac{d\rho}{dK}\bar{K} + a_2\right)^2 - 4a_1a_2\frac{d\rho}{dK}\bar{K}\right)^{1/2} \tag{68}$$

and

$$\lambda_{3;4} = \frac{1}{2\delta}\left(\frac{\partial \hat{\rho}_1}{\partial K}\bar{K}_1 + \rho_1\right) \pm \left(\frac{1}{4\delta^2}\left(\frac{\partial \hat{\rho}_1}{\partial K}\bar{K}_1 + \rho_1\right)^2 - 4\bar{C}_1\frac{\partial \hat{\rho}_1}{\partial K_1}\right)^{1=2} : \tag{69}$$

Because $\partial \hat{\rho}_1 / \partial K_1 < 0$, the last two eigenvalues are real numbers and can be ranked as follows:

$$\lambda_3 < 0 < \lambda_4 : \tag{70}$$

The first two eigenvalues are evaluated as follows:

$$\lambda_1 < 0 < \lambda_2 ; \tag{71}$$

if

$$\frac{d\rho}{dK} < 0 ; \tag{72}$$

and

$$\begin{aligned} \mathrm{Re}(\lambda_1) &> 0; \\ \mathrm{Re}(\lambda_2) &> 0; \end{aligned} \tag{73}$$

if

$$\frac{d\rho}{dK} > 0 : \tag{74}$$

Note that condition (72) is fulfilled in cases (i), (ii), and (iii) of proposition 2, whereas condition (74) is equivalent to the condition described in case (iv) of proposition 2 which follows from proposition 1.

Because the matrices A and B have the same characteristic polynomials (Lemma 2), the qualitative local stability properties are preserved by the transformation from the linnearization of system (64) to the linnearization of system (21)-(24). In particular, there exist two positive and two negative eigenvalues for system (21)-(24) in the cases (i), (ii), and (iii) of proposition 2 and 3 positive and 1 negative eigenvalue in case (iv). There correspond stable (unstable) eigenvectors to the stable (unstable) eigenvalues. By the stable manifold theorem, the local stable manifolds for the local steady states $x^*; x^{**}$; and x^{***} of cases (i), (ii), and (iii) are two-dimensional (i.e. a surface in R^4), whereas the local stable manifold for x^* in case (iv) is one-dimensional.

Next, it follows from case 1 in Buiter (1984) that a unique solution to the boundary value problem (21)-(26) exists and is stable in cases (i), (ii), and (iii), because the number of positive eigenvalues is equal to the number of control (jump-) variables $(C_1; C_2)$.

The boundary value problem (21)-(26) does not have a solution in case (iv), unless we give up one initial condition. Giving up the initial condition for $K_2(0) = K_{20}$ and letting K_2 "jump", yields again a unique and stable solution. If we inspect (68), we see that these are the eigenvalues of the dynamical subsystem (29) which determines convergence/non-convergence of K towards the steady state $\bar{K}$. If these eigenvalues are both positive, there will not be convergence of K. Therefore, we guess that the system (21)-(26) must be restricted in $K_2(0)$, such that relative capital ratios are in their steady state right from the beginning. Formally, we guess that

$$K_1(0) = K_2(0) = \tag{75}$$

will have to hold for any $\in R^{+}$. For any time $\tilde{t}$, there exists a $\tilde{\ } \in R^{+}$ such that $K_1(\tilde{t}) = \tilde{\ }$: By the property of autonomous differential equation systems, $\tilde{t}$ can be normalized to zero. Therefore, (75) implies that

$$K_1(t) = K_2(t) \tag{76}$$

for $t \geq 0$. From the first equation of (28) follows then that

$$C_1(t) = C_2(t) \tag{77}$$

for $t \geq 0$: Furthermore, from (16), (17), and (76) follows that $p(t) = 1$ for $t \geq 0$: The system (21)-(24) collapses to two independent neoclassical growth models. Therefore, the guess (75) is valid and yields indeed a stable solution to the boundary value problem (21)-(26) without the initial condition $K_2(0) = K_{20}$. Q.E.D.

Appendix 5: Proof of proposition 3.

We start out with equations (21) and (23). They can be integrated taken $C_j(0)$, and $\rho_j(t)$ to be well-defined (though unknown) functions of time t as given. (Note: $C_j(0)$ is to be solved for.)

$$C_j(t) = C_j(0)\, e^{\int_0^t (\rho_j(s) - \lambda) ds} \tag{78}$$

Integrating in the same way (22) and (24) yields

$$K_j(0) = \int_0^\infty C_j(t)\, e^{-\int_0^t \frac{1}{\theta}\rho_j(s) ds} dt; \tag{79}$$

where we made use of the intitial condition (25) and the transversality condition (26). Plugging (78) into (79) yields:

$$C_j(0) = \mu_j(0)\, K_j(0); \tag{80}$$

where

$$\mu_j(0) = \left(\int_0^\infty e^{-\int_0^t \left(\frac{1-\theta}{\theta}\right)\rho_j(s) + \lambda\, ds} dt \right)^{-1}:$$

(The three steps are standard in the literature, e.g. Barro and Sala-i-Martin (1995), p. 59ff., in a similar model.) Suppose now

$$\text{(i)} \qquad K_{10} = K_{20} = \tilde{\ };$$

with any $\tilde{\ } \in R^{+}$. Suppose further that with this initial condition the steady state x^{**} will be reached, i.e.:

$$\text{(ii)} \qquad \bar{K}_1 > \bar{K}_2;$$

i.e. $\bar{K} = K^* < 1$: Finally, we assume without loss of generality that $K(0) \quad K(s) \quad K^*$ for $0 \quad s \quad 1$.[48] From this assumption and proposition 1 follows that

$$\rho_1(s) \geq \rho_2(s) \tag{81}$$

[48] In other words: If $K_1(s) = K_2(s)$ for $s \in \{s', s'', ...\}$, and $s' \geq s'' \geq ...$ on the same trajectory reaching x^{**} (if it exists), then we normalize by the property of autonomous systems $s' = 0$. Then follows that $K_1(s) > K_2(s)$; i.e. $K(s) < 1$, for $s > 0$, because $s = 0$ is the last point in time, where $K_1(s) = K_2(s)$ is sustained and before the steady state $K^* < 1$ is reached. Furthermore, x^{**} is a stable node which follows from the proof of proposition 2. Therefore, the steady state value is not "overshooted" (as would be the case for a stable focus), i.e. $K(s) \geq K^*$ for $s > 0$.

for $0 \le s \le 1$ and a strict inequality for some s. Consequently,

$$\left(\frac{1-\delta}{\delta}\right)\rho_1(s)+\lambda \ge \left(\frac{1-\delta}{\delta}\right)\rho_2(s)+\lambda \tag{82}$$

for $0 \le s \le 1$ and a strict inequality for some s. One may check that this implies

$$\mu_1(0)=\left(\int_0^{\infty} e^{-\int_0^t\left(\frac{1-\delta}{\delta}\right)\rho_1(s)+\lambda\, ds}dt\right)^{-1} > \left(\int_0^{\infty} e^{-\int_0^t\left(\frac{1-\delta}{\delta}\right)\rho_2(s)+\lambda\, ds}dt\right)^{-1} = \mu_2(0) \tag{83}$$

and therefore by equation (80) and assumption (i)

$$C_1(0) > C_2(0): \tag{84}$$

However, then follows from (28) and assumption (i) that

$$\dot{K}_1(0) < \dot{K}_2(0); \tag{85}$$

i.e. $\dot{K}(0) > 0$. Recall that $K(0) = 1$ (assumption i) and $K^* < 1$. Therefore, the direction of movement will always point away from the steady state K^*, if $K(0) = 1$: By the properties of an autonomous differential equation system, the trajectory to the steady state can never pass the threshold $K = 1$ at any point in time in the direction of the steady state and therefore not reach the steady state. This contradicts assumption (ii). Therefore, there is no perfect foresight path from the initial condition $K(0) \ge 1$ to the steady state K^*: By the symmetry property of the model, there is also no perfect foresight path from the initial condition $K(0) \le 1$ to the steady state $(1/K^*)$: From proposition 2 case (iv) follows that there exists a one-dimensional stable manifold such that $\bar{K} = 1$ is reached, if $K(0) = 1$. This concludes the proof. Q.E.D.

Appendix 6: Proof of Proposition 4.

A sufficient condition for $U_2^{**}(q) < U_1^{**}(q)$ is that

$$C_2^{**}(t;q) < C_1^{**}(t;q); \qquad 0 \le t \le 1 \tag{86}$$

(see equation 30). $C_j^{**}(t;q)$; $j = 1;2$; is given analogously by equation (80) in appendix 5. From (81)-(84) of appendix 5 follows analogously that a set of sufficient conditions for (86) are

$$K_{2t} < K_{1t}; \qquad 0 \le t \le 1 \tag{87}$$

and

$$\rho_{2t} < \rho_{1t}; \qquad 0 \le t \le 1: \tag{88}$$

Condition (87) follows from the properties of the poverty trap regime as has been found in proposition 3. If (87) holds, then (88) holds in the poverty trap regime ($q < q^*$) by Lemma 1. Hence, $U_2^{**}(q) < U_1^{**}(q)$; for $q < q^*$. Q.E.D.

Appendix 7: Proof of Proposition 5.

We first define $p^{**}(t;q)$; and $n^{**}(t;q)$ as the trajectory of the ratio of producer prices of country 2 relative to country 1, and the ratio of the number of firms of country 2 relative to country 1 from a given initial condition K_{10} t K_{20} towards the steady state x^{**}, respectively. Correspondingly, we define $p^*(t;q)$; and $n^*(t;q)$ as the trajectory of the ratio of producer prices of country 2 relative to country 1, and the ratio of the number of firms of country 2 relative to country 1 from a given initial condition

$K_{10} \gtrless K_{20}$ towards the steady state x^*, respectively. Finally, we define $\pi_2^*(t;q)$ and $\pi_2^{**}(t;q)$ as the trajectories of the ratio of the producer price of country 2 relative to the consumption price index of country 2, if steady state x^* and x^{**} are approached, respectively. Then, we can formulate a useful lemma.

Lemma 2: $\pi_2^*(t;q) > \pi_2^{**}(t;q)\,;\ 0 \le t \le \infty\,;$ for a given q, $q < q^*$; and a given initial condition, $K_{10} \gtrless K_{20}$.

Proof: First, note that

$$n^{**}(t;q) < 1 \tag{89}$$

for all t and a given $q < q^*$ by proposition 3 and

$$p^{**}(t;q) < 1 \tag{90}$$

for all t and a given $q < q^*$ by (89) and Lemma 1. Second, note that

$$\pi_2^*(t;q) = \left(\frac{1+q}{2}\right)^{\frac{1}{\sigma-1}}; \tag{91}$$

and

$$\pi_2^{**}(t;q) = \left(\frac{qp^{**}(t;q)^{\sigma-1} + n^{**}(t;q)}{1+n^{**}(t;q)}\right)^{\frac{1}{\sigma-1}}. \tag{92}$$

Then, we define a function $\hat{\pi}_2^{**}(n^{**}(t;q)\,;t;q)$ as:

$$\hat{\pi}_2^{**}(n^{**}(t;q)\,;t;q) \equiv \left(\frac{q+n^{**}(t;q)}{1+n^{**}(t;q)}\right)^{\frac{1}{\sigma-1}}. \tag{93}$$

Next, we take the partial derivative of this newly defined function with respect to the relative number of firms:

$$\frac{\partial\,\hat{\pi}_2^{**}(n^{**}(t;q)\,;t;q)}{\partial n^{**}(t;q)} = \left(\frac{1-q}{1+n^{**}(t;q)}\right)\left(\frac{1}{\sigma-1}\right)\left(\frac{q+n^{**}(t;q)}{1+n^{**}(t;q)}\right) > 0. \tag{94}$$

Since $\hat{\pi}_2^{**}(1;t;q) = \pi_2^*(t;q)$, it follows under the condition (89) and the result (94) that

$$\hat{\pi}_2^{**}(n^{**}(t;q)\,;t;q) < \pi_2^*(t;q)\,. \tag{95}$$

However, it is true by inspection of (92) and (93) that

$$\hat{\pi}_2^{**}(n^{**}(t;q)\,;t;q) > \pi_2^{**}(t;q)\,; \tag{96}$$

where we note (90), and $\sigma > 1$ by assumption. Hence, we can conclude from (95) and (96) that

$$\pi_2^*(t;q) > \pi_2^{**}(t;q)\,. \tag{97}$$

Q.E.D.

Now, we are ready to begin the proof for proposition 5.

Begin of Proof:

Fix a q; $q < q^*$. Then, there exists an equivalent optimization problem to (21)-(26) yielding the same trajectory for consumption in country 2, $C_2^*(t;q)$; in the symmetry growth regime approaching steady state x^*, if (1) is maximized with respect to C_{2t}, subject to the transversality condition (25), the initial condition (26), and the intertemporal budget constraint

$$\dot{K}_2 = \pi_2^{*}(t;q)\, K_2^{\delta} - C_2; \tag{98}$$

where the time index is suppressed whenever obvious and $\pi_2^{*}(t;q)$ is defined on top of appendix 7.

Analogously, there exists an equivalent optimization problem to (21)-(26) yielding the same trajectory for consumption in country 2, $C_2^{**}(t;q)$; in the poverty trap regime approaching steady state x^{**}, if (1) is maximized with respect to C_{2t}, subject to the transversality condition (25), the initial condition (26), and the intertemporal budget constraint

$$\dot{K}_2 = \pi_2^{**}(t;q)\, K_2^{\delta} - C_2; \tag{99}$$

where $\pi_2^{**}(t;q)$ is defined on top of appendix 7. Then, define a sub-optimal consumption path $\hat{C}_2(t;q)$ in the symmetry growth case such that

$$\hat{C}_2(t;q) \equiv \pi_2^{*}(t;q)\, K_2^{**}(t;q)^{\delta} - \dot{K}_2^{**}(t;q); \tag{100}$$

where $K_2^{**}(t;q)$, and $\dot{K}_2^{**}(t;q)$ are the optimal trajectory of the capital stock of country 2 in the poverty trap case approaching the steady state x^{**}, and its time derivative, respectively. It holds that

$$\hat{C}_2(t;q) - C_2^{**}(t;q) = [\pi_2^{*}(t;q) - \pi_2^{**}(t;q)]\, K_2^{**}(t;q)^{\delta} > 0; \qquad 0 \le t \le \infty; \tag{101}$$

where the inequality sign follows from Lemma 2 of appendix 7. Consequently, the following inequality holds:

$$\hat{U}_2(q) \equiv \int_0^{\infty} e^{-\lambda t} \ln \hat{C}_2(t;q)\, dt > \int_0^{\infty} e^{-\lambda t} \ln C_2^{**}(t;q)\, dt \equiv U_2^{**}(q): \tag{102}$$

The two trajectories $\hat{C}_2(t;q)$ and $K_2^{**}(t;q)$ are feasible for the program (1), (25), (26), and (98) by construction of $\hat{C}_2(t;q)$ in definition (100). However, these trajectories are not optimal for this program, because $K_2^{**}(t;q) \neq K_2^{*}(t;q)$ for at least $t = \infty$, and because there exists only one optimal perfect foresight path for a given set of initial conditions and a given q by proposition 3. Hence, it must be true that

$$\hat{U}_2(q) < U_2^{*}(q): \tag{103}$$

Finally, it follows from (102) and (103) that

$$U_2^{**}(q) < U_2^{*}(q): \tag{104}$$

q.e.d.

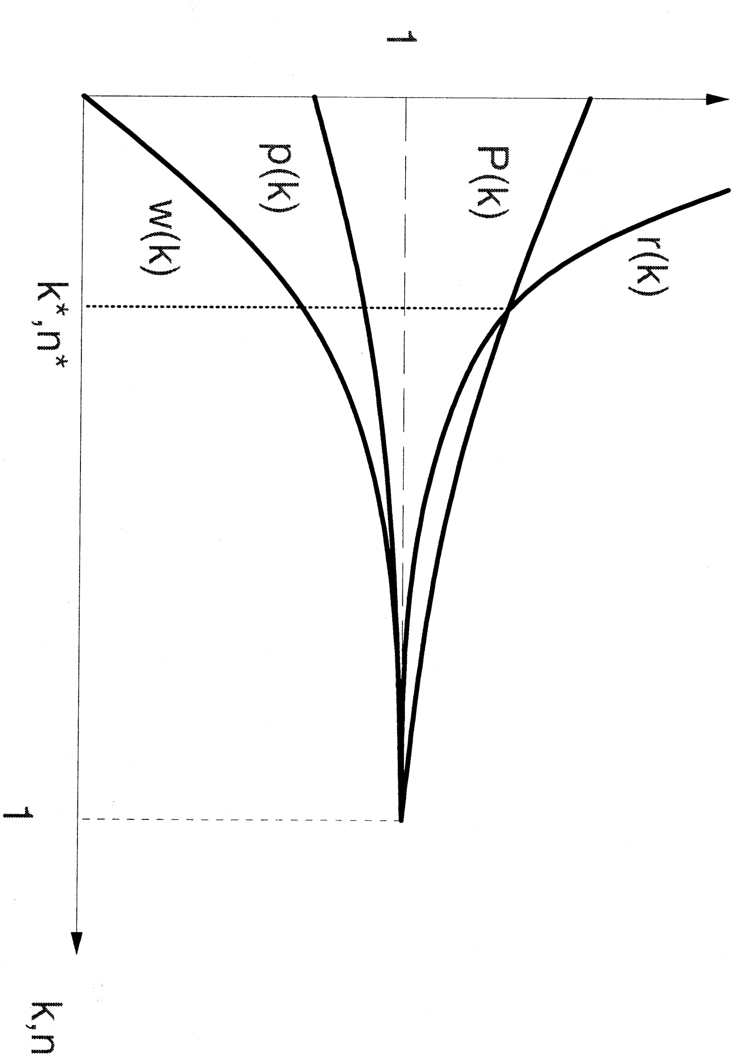

Figure 1: Relative prices, wages, and rental rates

Figure 1:

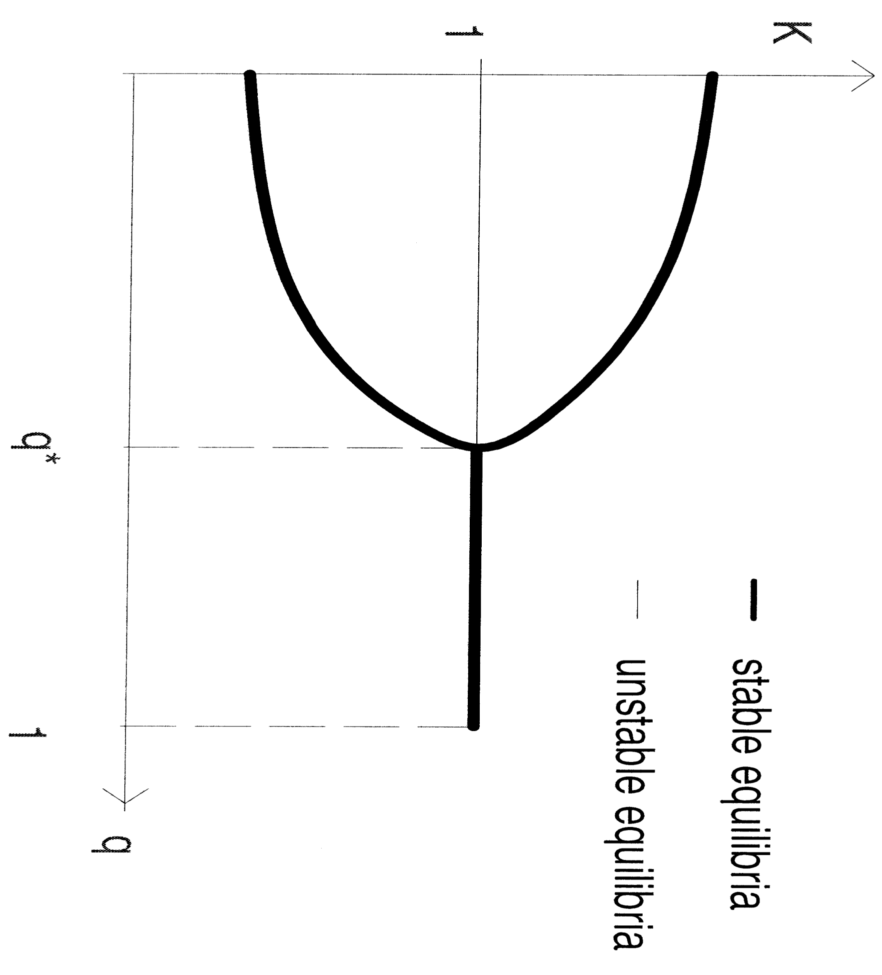

Figure 2: Bifurcation Diagramm

Figure 2:

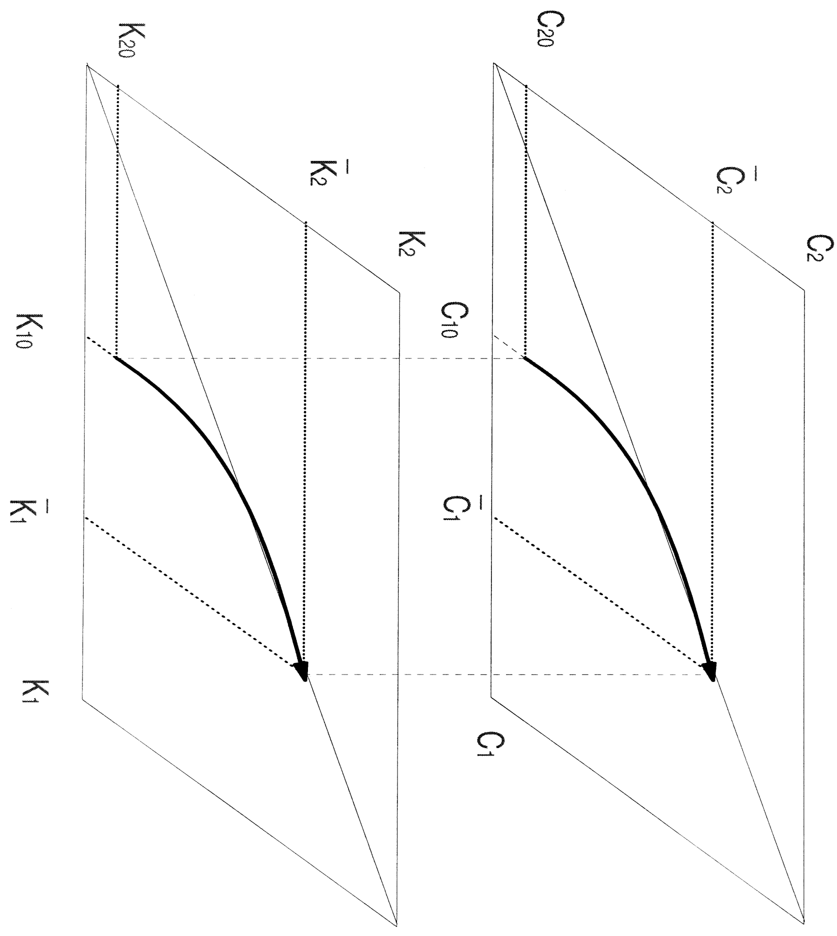

Figure 3: The Neoclassical Regime

Figure 3:

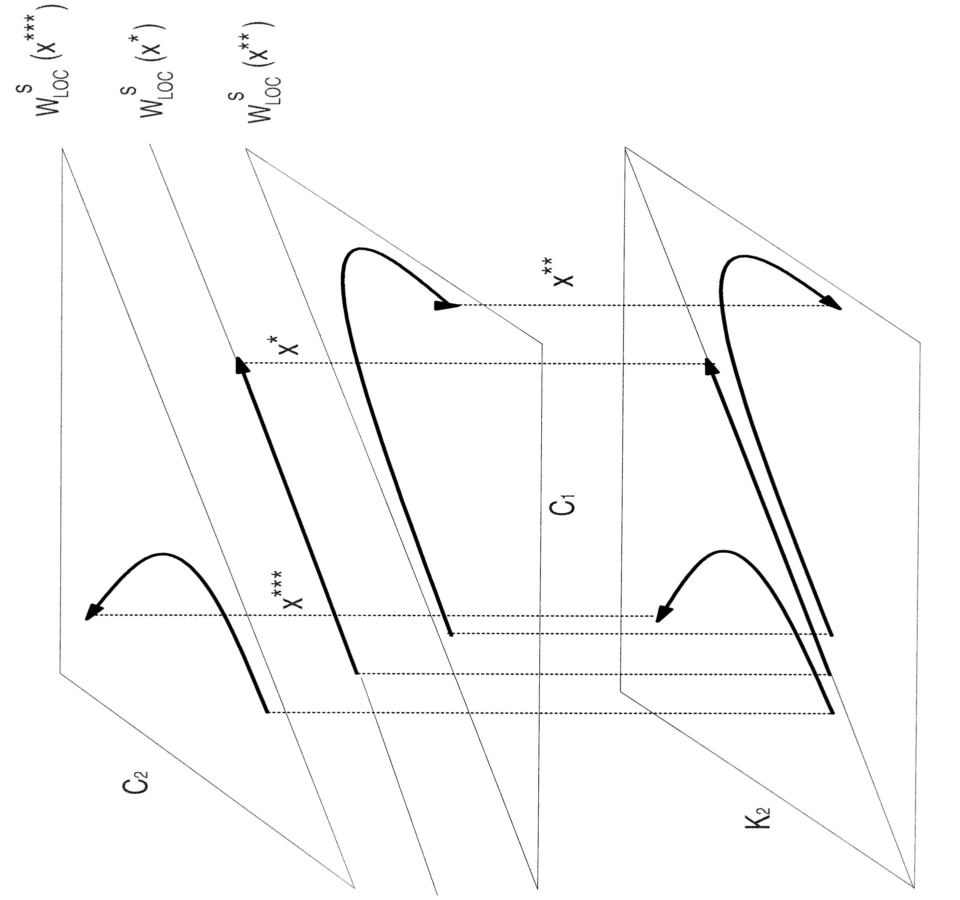

Figure 4: The Poverty Trap Regime

Figure 4: