Wealth Constraints, Sharecropping Contracts and Transition to a Modern Sector

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Abstract

In a traditional agrarian sector tenancy contracts allow farmworkers to produce and accumulate wealth. This is important in a dual economy characterized in its modern sector by a high degree of credit imperfections. The wealth constrained workers can accumulate wealth by working as sharecroppers so that the next generation is able to invest in the modern, more efficient sector. We show that production in the traditional sector influences the modern sector and vice versa. Therefore the impact of both agrarian reforms and monetary wealth redistributions have to be evaluated regarding both the sectors and considering the possibility of intersectorial migration. We will see that for land poor economies agrarian reforms have an impact on the aggregate production while for land rich economies this can be totally ineffective. Furthermore there are equilibria where monetary wealth redistributions are preferred to the agrarian reforms.

Keywords: Sharecropping contracts, credit imperfections, wealth accumulation, agrarian reform, monetary wealth redistribution, dual economy, intersectorial migration.

JEL Classification: O12, O14, O20, Q15

1 Introduction

In a perfect world without wealth constraint all individuals choose the most profitable investments and, to the extent that there is perfect information, a technology in the production frontier is always chosen. Wealth constraints represent an important obstacle
for the full production efficiency and the economic growth, and the more imperfect the credit markets are, the less efficient the economic exploitation of both physical and human resource. Many contributions (see Legros and Newman (1997) for a theoretical analysis in a general equilibrium framework of asset allocation and wealth constraints) have emphasized that in a less developed economy, where the credit markets are particularly imperfect or even non-existent, this is a fundamental barrier to the modernization of the economy.

In the traditional rural economy, the social organization and, more particularly, the nature of human relationships allows agents to overcome this problem. The literature has already pointed out that in traditional societies, given the particularly close nature of their relationship, individuals can better enforce credit contracts and overcome their wealth barriers.

In a recent paper Banerjee and Newman (1998), considering a dual economy, showed that this possibility of easy credit in the traditional economy can hamper the process of transition to a modern one. They focus on the consumption credit which is easier to obtain in the rural villages than in a more impersonal, urban economy. They claim that an easy credit access in the traditional society makes it more appealing for individuals to stay in the village rather than migrating toward more efficient and modern ways of production.

In this paper we consider this aspect on the “supply side”, namely that individuals have an easier access to the production factors (in our case the land) in the rural rather than in the modern economy, and we claim that this can be positive for the transition of the whole economy. More precisely, we consider that poor dynasties who have the possibility of producing in the rural economy can allow their future generations to migrate toward more modern sectors. In order to characterize the traditional sector we model a general tenancy contract and we show how the sharecropping contracts are an endogenous agreement which arises when workers are wealth constrained and the quantity of land per worker is low. We then analyze the role of this traditional kind of agreement in promoting the economic development.

In order to endogenize the terms of tenancy contracts, we set a simple general equilibrium model of a dual economy and we analyze how initial conditions on wealth and land distribution affect the efficiency of the economy in the long run equilibrium. To determine long run equilibria, we perform an exercise of intergenerational wealth accumulation and we investigate the link between tenancy contracts and the size of the modern sector. This type of exercise allows us to consider the role of both the agrarian reform (which in this paper we consider as a land redistribution) and the wealth redistribution in promoting the modernization of the economy and the efficiency of the agrarian sector. We will show that two kinds of long term equilibria are possible.

The equilibrium with “low modernization” where all tenancy contracts are ineffi-
cient. In this case the high level of land demand causes a high level of rent and a long run inefficiency in the agrarian production. The land ownership in this case is important for the dynasties because: i) it allows them to produce more efficiently when they are in the traditional sector; ii) it represents an “insurance” for the absentee owners’dynasties who can finance their investment in the modern sector with the revenue from their rented lands. Therefore, even when these dynasties are not successful in the modern sector they can still supply wealth to their offsprings and allow them to invest in the modern sector. In this case an agrarian reform has the positive effect of increasing the number of dynasties that benefit from these two effects.

In the equilibrium with “high modernization” all the tenancy contracts are first best efficient because there is not rent from the land. Given this efficiency the quantity of individuals in the modern sector is maximum. In this case both the wealth and the land distribution cannot have any impact on the two sectors. Therefore a policy of wealth redistribution and an agrarian reform cannot improve the efficiency in the economy. The economy reaches this kind of equilibria when the quantity of land per person is high.

Furthermore we point out that in some cases it is possible with a wealth distribution or with an agrarian reform to move the economy from the first to the second equilibrium. In that way it is possible to improve the modernization of the economy and the efficiency of the tenurial contracts. Finally we will show that for some given initial conditions only one of the two policies has an impact on the long run equilibrium.

The importance of wealth distribution on the modernization of the economy seems to be supported by Alesina and Rodrik (1994) who showed that the index of land concentration has a significant negative effect on the growth rate of the economy. Their theoretical argument was that a higher level of inequality hampers growth because individuals will vote for more inefficient redistribution policies, and the land distribution is used as a proxy for the wealth. Nevertheless, in their subsequent empirical analysis the democratic countries are not more sensitive to the coefficient of land concentration than the dictatorships. This leads them to conclude that this negative coefficient might be due to a different explanation.

There is also some empirical evidence of the result that agrarian reforms did not have an important positive impact on the “land rich”economies. As Moene (1994) pointed out, agrarian reforms tend to be more successful in regions where the quantity of land per capita is lower. To give an example, in China and in Taiwan in which their respective land reforms are traditionally considered successful, the crop land per capita is 0.10 and 0.06 hectares respectively. On the other hand, the land reforms of Mexico and Peru do not seem to have had any important impact on their respective economies, and the crop land per capita of these countries is respectively 0.31 and 0.19 hectares.

1.1 Literature review

The observation that wealth constrained individuals need to accumulate wealth before realizing any projects is present in a paper of Gathak, Morelli and Sjöström (1997).
In this paper this necessity might be a positive incentive for young individuals to work harder (American dream effect). The fundamental difference with our paper is that they consider that the modern sector is always able to give a job to young people to enable them to accumulate wealth, whereas in our paper the only way to accumulate wealth is to work in a traditional sector. The idea of the American dream effect can better apply to an already developed economy, while our paper can be more usefully related to a less developed economy.

Our work is also related to the literature on wealth distribution and growth. In particular Aghion and Bolton (1997) emphasize the same idea that a more egalitarian wealth distribution allows more individuals to invest when the production activity requires an initial wealth level. The difference is that they consider only one sector in which there is a positive externality in producing more wealth because this increases the supply of funds for investments. Consequently in Aghion and Bolton’s paper the distribution of wealth always has the effect of improving this process, while in our model for some initial conditions wealth and land distribution does not have any effects.

Another contribution which focuses on the effect of wealth distribution and growth is Banerjee and Newmann (1993). They show that if wealth distribution is not too unequal the economy ends up in an equilibrium of “prosperity”. Vice versa, when there are a lot of poor and only few rich, the economy can fall in a poverty trap equilibrium. Also in this paper the role of the land market and of the agrarian reform is not considered.

The problem of the agrarian reform is often related to the problem of the efficiency of sharecropping contracts. In an extremely simplifying way we can distinguish two different considerations which have very different political economy implications. The first consideration is that workers in the agrarian sector are poor and the information asymmetries are particularly pervasive, therefore sharecropping contracts are considered as a second best way to provide the land for the farmers. The second consideration is that this is based on a difference of skills or attitude toward the risks of landlords and farmers, therefore sharecropping can result in being an optimal device for labor division or risk sharing.

In the first approach the landlord obtains an unproductive rent, so doing away with him is only an improvement. In the second approach the problem is to consider how important the role of the landlord is in the production process and whether the farm workers, who become richer after the land reform, can substitute him.

The present paper belongs to the first framework (even if in the conclusive section we will do some considerations related to the second line). Nevertheless, we not only consider the relation between agrarian reform and agrarian sector as it is generally done

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3 For a survey on this argument see Banerjee (1998)
4 See Banerjee, Gertler and Gathak (1998)
5 See Eswaran and Kotwal 1985
Figure 1:

in literature, but we also are interested in the influence the traditional agrarian sector has on the modern one.

This same relationship is considered in Moene (1992). As in our paper, he considers the impact of land demand and supply on the rent and on the efficiency in production, but he does not consider the role of the agrarian sector in the modernization of the economy and he focuses more on the effect on urban and rural poverty. Furthermore, in his paper the wealth of individuals has an important impact on agrarian production, but does not have effect on the possibility of being employed in the modern sector.

2 The Model

We consider an economy with a continuum of risk neutral agents of total mass 1. In this section and in the next we take a one period model, from third section on we will consider the dynamic model with infinite times. Production can take place both in a modern industrial and in a rural agrarian sector. We will call these two sectors respectively M-sector and T-sector. In T-sector individuals produce by using a plot of land and effort, in the M-sector capital is the only production factor. The set of plots of land is a continuum and has a measure \( \Omega \). We suppose that a measure \( \Omega \leq L \) of the land is owned by \( O \) individuals (hereafter owners). The rest is owned by another class of individuals that we will refer to as “landlord”. We suppose that the number of these individuals is finite and they are more than one. They are not able to supply any effort. The land distribution is depicted in figure 1.

The rest of \( 1 - O \) individuals (that we will call hereinafter “Workers”) can produce in the T-sector by renting a plot of land either from a landlord or from an absentee owner. The output for each plot of land is uncertain and the revenue is given by:

\[
q^T = \begin{cases} 
1, & \text{with probability } e \\
0, & \text{with probability } 1 - e 
\end{cases}
\]

(1)

The probability of success is influenced by an unobservable individual effort \( e \). For computational simplicity, let us suppose that the cost of effort has the following functional form: \( \frac{e^2}{2\sigma e} \). It is negatively correlated to the exogenous technological parameter
\( \alpha_T \in (0, 1) \) therefore his optimal level will be a function \( e^x(\alpha_T) \). Consequently the probability of being successful is a function of the effort and of the exogenous parameter \( \alpha_T \).

Furthermore, let us suppose that beside its value as production factor, each single plot of land gives to its owner a non monetary utility \( u \). We can think of this value as social prestige of owning land.

In the M-sector we abstract from every agency problems and the probability of success is only determined by a completely exogenous idiosyncratic shock with probability \( \alpha_T \). Accordingly, they invest a fixed amount \( k \) and they obtain:

\[
q_T^* = \begin{cases} 
  y, & \text{with probability } \alpha_N \\
  0, & \text{with probability } 1 - \alpha_N 
\end{cases}
\]  (2)

Where \( y \in \mathbb{R}^+ \) is an exogenous technological parameter. We suppose that an individual in the M-sector can always “runaway” after the production takes places and without risk of being caught. Therefore there is not credit market in this sector.

The success in the T-sector is determined by the level of effort and the exogenous parameter \( \alpha_R \), in the M-sector the determinant of the positive result is only the exogenous parameter \( \alpha_N \). For simplicity we can suppose that the idiosyncratic shock is common to the overall economy and consequently that \( \alpha_T = \alpha_M = \alpha \). This value gives a measure of the aggregate productivity of the economy. Alternatively we can think that its value is cyclical and depends on the economic conjuncture.

Individuals achieve maximum utility in the T-sector when they obtain ex post the total production. In this case the expected utility is \( \frac{y}{2} \). We suppose that the M-sector is more efficient than the T-sector so that if all individuals were not wealth constrained they would prefer to produce in the M-sector. Therefore we make the following fundamental assumption:

**Assumption 1** *The profit in the M-sector is larger than the maximum profit that an individual can obtain in the T-sector, accordingly \( y > \frac{1}{2} + \frac{1}{\alpha} \).*

### 2.1 Wealth and land distribution

From assumption 1 only individuals with wealth \( w < k \) produce in the T-sector. All poor individuals produce in their own land if they are owners, otherwise they can demand a land on the rental market and sign a contract with a landlord or an absentee (rich) owner. Given a wealth distribution and a land distribution we can determine the land supply and demand in the land rental market. The land supply is equal to all lands

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\( ^7 \) Parameters \( \alpha_R \) an \( \alpha_N \) when different allow to consider separately redditivity and risk of any investment. In the present paper we abstract from this kind of consideration since we suppose that all individuals are risk neutral.
owned by landlords plus the land owned by the individuals with \( w \geq k \), while the land demand is equal to the mass of workers with wealth \( w < k \).

We start with a very simple bivariate wealth-land distribution. Like we already saw, agents are different according to whether they are owners or not, furthermore we suppose that individuals can be either “monetary poor” with 0 wealth or “monetary rich” with wealth \( w = \bar{w} \geq k \).

Therefore we have the following land-wealth distribution:

1. \((1 - O)(1 - \omega^o)\) agents with no land and wealth 0.
2. \((1 - O)\omega^w\) agents with no land and wealth \( \bar{w} \).
3. \(O(1 - \omega^o)\) agents with land and wealth 0.
4. \(O\omega^o\) agents with land and wealth \( \bar{w} \).

Since we supposed that \( \bar{w} \geq k \), all rich agents can produce in the M-sector (owners and not). Furthermore let us suppose that \( k > \frac{u}{k} + u \), this rules out the possibility for the owners to sell the land and produce in the M-sector. Finally, let us assume that \( \omega^o \geq \omega^w \). In the next section we will endogenize these terms and we will see that when wealth accumulation is endogenous it will never be possible that \( \omega^o < \omega^w \).

### 3 Static equilibria and tenancy contracts

Rich owners and landlords supply the land in the rental market, accordingly the land supply is:

\[ S(L, O, \omega^o) = L - O(1 - \omega^o) \quad (3) \]

All poor workers demand for land then:

\[ D(M, \omega^w) = (1 - O)(1 - \omega^w) \quad (4) \]

Comparing (3) and (4) we have the following:

#### 3.1 Tenancy contract

Let us start to consider the case where there is an excess of land demand: \( S(L, O, \omega^o) < D(O, \omega^w) \). Landlords and owners have all the bargaining power and the value of the outside option for the poor workers is 0. Since workers have wealth 0 they can only pay ex-post the total amount needed to obtain the land.

The value of the ex-post payment and the decision whether to rent or to sell the land is entirely chosen by landlords and owners who want to maximize their expected utility:
\[
\max_{e,v|e,v \in \{0,u\}} e(s,v)s + v \tag{5}
\]

where \(v_1 = 0\) if owners or landlords sell the land; \(v_1 = u\) if they rent it and, therefore, still enjoy the non monetary utility \(u\). The level of \(e\) is not observable therefore workers will:

\[
e(s,v) \in \text{Arg} \max e(1-s) - \frac{e^2}{2\alpha} \tag{6}
\]

Maximizing (5) in the set 6 it is easy to see that: \(s^* = \frac{1}{2}\) : landlords and owners prefer to leave half of the production to the workers in order not to disincentive them too much. Since the workers will obtain only half of the production, there will be a distortion and the level of effort is suboptimal and equal to \(\frac{e}{T}\). Moreover \(v^*_w = u\) : landlords and owners prefer to keep the ownership and not to sell the land because, in order to give better incentive to the workers, they prefer that their payment is totally contingent to the final result (if they had sold the land for an ex-post payment then workers would have obtained the utility \(u\) independently from the final outcome).

We consider now an excess of land supply: \(S(L,O,\omega^o) > D(O,\omega^m)\). In this situation workers have all the bargaining power:

\[
\max_{v,w \in \{0,u\}; \ast \epsilon} e(1-s) - \frac{e^2}{2\alpha} + v_w \tag{7}
\]

In Appendix we see that: \(s^* = 0\) and \(v^*_w = 0\). Workers prefer not to buy the land because they do not want to pay any ex-post amount. This happens because they are fully residual claimants and can obtain all the surplus deriving from their effort \(e\). Therefore if they have to pay an amount equal to the utility \(u\) ex-post, their level of effort will not be optimal anymore and they will entirely pay the inefficiency of the contract.

Finally, when \(S(L,O,\omega^o) = D(O,\omega^m)\), we have a situation of bilateral monopoly with the level of expected rent which ranges in the interval \([0, \frac{1}{2}]\). The right limit of the interval being the expected rent when \(s = \frac{1}{2}\).

Considering what we said we have the following:

**Lemma 1** When \(L \leq 1 - O(\omega^o - \omega^m) - \omega^o\) there is excess of land demand and the level of effort is suboptimal. When \(L > 1 - O(\omega^o - \omega^m) - \omega^o\) there is excess of land supply and the level of effort is first best efficient \(e^* = \alpha\).

This lemma points out that in our model the terms of the tenurial contract depend endogenously on the wealth and land distribution determined by the vector \(\{\omega^o;\omega^m;O;L\}\)
As we saw, the reason is simple: rich individuals prefer to migrate to the modern sector and this determines the land supply and demand. Therefore we can argue that in order to consider the efficiency of the tenancy contracts it is important to take account of the intersectoral migration between sectors and, consequently, the wealth-land distribution across individuals.

More in general, this lemma is an application to a developing economy of the concept expressed by Legros and Newmann (1997). They show that in a general equilibrium framework, when there are wealth constraints efficiency of the contract depends on wealth distribution. In the next section we endogenize the wealth distribution with a simple dynamic of intergenerational accumulation. Long run equilibria will depend on the production level in the two sectors and for lemma 1 on the wealth distribution in the preceding periods. Therefore we will determine the initial conditions which lead the economy to the different equilibria. Finally we show how and when the economic policies of redistribution improve the total efficiency of the economy.

4 Long Run Equilibria

Let us suppose now that every individual has one offspring and individuals reproduce themselves infinitely. In order to model the process of wealth accumulation in the easiest possible way, we need to do some specific assumptions on the timing the agents invest and consume and on their preferences.

4.1 The dynamic model

Let us suppose that the life of every individual is subdivided in two periods. In the first period individuals work, invest and consume, in the second they obtain the return on their investment and split their wealth in consumption and bequest. We assume that the utility function has the form:

$$c_1 + Min\{\delta c_2; (1 - \delta)b\} - \frac{e^2}{2\delta}$$

(8)

Where $c_1$ and $c_2$ are the consumptions in the two subperiods and $b$ is the bequest the individuals leave to their offsprings ($\eta = \delta(1 - \delta)$ is a factor of normalization). Given this function, in the second period the indirect utility function for all individuals is $\delta \eta$, for computational simplicity we multiply the cost function of effort by a factor $\delta$, in that way the level of effort will be the same as in the preceding section. Moreover, Assumption 1 has to be slightly modified as:

**Assumption 1 bis** $y > \frac{1}{2} + \frac{\delta}{\eta}$
In the first subperiod of time $t$ individuals invest to maximize the output $q$ under the wealth constant $i + c_1 = b_{t-1}$, where $b_{t-1}$ is the bequest inherited. Furthermore, the process of wealth accumulation will depend on the level of the rent, for both the owner and the workers. Accordingly it will be:

$$b_t = \delta q_j (b_{t-1}, \rho)$$  \hspace{1cm} (9)

Where the index $j$ can be: “Owner” or “Worker”. Like we saw $\rho$ depend on the demand end supply and, consequently, on the wealth distribution. For this reason the process of wealth accumulation is not stationary. For this reasons is impossible to use the standard techniques (see Stockey Lucas 1989) and determine a limit wealth distribution. Therefore we will describe this dynamic under the two possible situations of excess of supply and excess of demand at time $t - 1$. In that way we find the level of demand and supply at time $t$ and we can always see in which of the two market equilibria (excess of supply or demand) the economy is.

The choice of investing in the M-sector depends on the wealth accumulation. From the previous section we know that all individuals with wealth $w \geq \bar{w}$ prefer to invest in the M-sector. Therefore, given our assumptions, all individuals who inherited a bequest $b \geq \bar{w}$ will prefer the above sector.

Given that individuals do not save the not invested wealth, the bequest will always be a portion of the production. When there is excess of land demand, the wealth that a successful worker in the T-sector bequeaths is $\frac{b}{T}$. Therefore, in order to allow individuals that are not owners to change sector, we assume that the level of accumulation is high enough, then:

Assumption 2 $\frac{b}{T} \geq k$

4.2 The rental market in the dynamic model

Given assumption 2, the dynamics of intersectorial migration between the two sectors becomes quite simple and we can compute the measure of all dynasties of individuals who are in the M-sector in both the market equilibria (excess of supply and excess of demand), we will consider this measure as an index of modernization of the economy.

4.2.1 Excess of land demand

Let us start by considering the case of demand larger than supply at time $t - 1$. The contract that arises is a pure sharecropping contract in which either the landlords or the absentee owners agree to split the final surplus with the workers in equal parts. Given Assumption 2, dynasties of successful sharecroppers and of absentee owners whose sharecropper was successful will work in the M-sector, given that they inherited $\frac{b}{T}$.

The migration between $t - 1$ and $t$ of each dynasty of owners is described in the following table:
At time \( t \) in the T-sector there are all the dynasties of owners who at time \( t - 1 \) produced unsuccessfully in the same sector and who rented the land to an unsuccessful tenant. Given that we are considering a continuum of individuals, the probability of being successful can be considered as the proportion of successful workers on the total, therefore the mass of owners in the T-sector is described by the following equation:

\[
o^T_t = (1 - \alpha)o^T_{t-1} + (1 - \frac{\alpha}{2})(1 - \alpha)o^M_{t-1}
\] (10)

Where with \( n^i \) we indicate the total amount of owners in the sector \( i \). Since the term \( \frac{\alpha}{2} \) represents the probability of being successful for a sharecropper, the second term of (10) represents the total amount of owners who came from the M-sector.

At the same time in the M-sector there are all dynasties of owners who were successful at \( t - 1 \) plus the unsuccessful dynasties whose tenant was successful, accordingly:

\[
o^M_t = (\alpha + (1 - \alpha)\frac{\alpha}{2})o^M_{t-1} + \alpha o^T_{t-1}
\] (11)

Let us consider now the workers. The behavior of their dynasties is depicted in the following table:

<table>
<thead>
<tr>
<th>( t, t-1 )</th>
<th>Demand for land</th>
<th>M-sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-sector</td>
<td>Unsuccessful</td>
<td>Successful or Unsuccessful and tenant Successful</td>
</tr>
<tr>
<td>M-sector</td>
<td>Successful</td>
<td>Successful or Unsuccessful and tenant Successful</td>
</tr>
</tbody>
</table>

In the M-sector there are all the successful dynasties of the preceding periods, accordingly the workers into the M-sector are:

\[
w^M_t = \alpha w^M_{t-1} + \frac{\alpha}{2} w^T_{t-1}
\] (12)

Given that \( \alpha \) is the probability of success in the M-sector and \( \frac{\alpha}{2} \) is the probability of success for a sharecropper. The equation of dynasties of workers in the T-sector can be derived residually recalling that we are considering the case in which at \( t - 1 \) the land demand was larger than the supply. Consequently, all the land is used by the owners and the workers and the following equality has to be true:

\[
w^T_t = L - o^T_{t-1}
\] (13)

In order to find the equations solutions of these last three equations we can solve (10) for \( o^M_{t-1} \) and substitute \( o^M_t \) and \( o^M_{t-1} \) in (11). In that way we have the following second order homogenous difference equation:
In appendix we solve this equation considering that the economy starts from a given wealth and land distribution \( \{ \omega_0^M; \omega_0^T; O_0; L \} \).

We substitute the equation for \( \omega_{t+1}^T \) in (13) and we have the equation for \( u_{t-1}^T \). Substituting this last expression (see Appendix) in (12) and iterating we have the dynamic equation for \( u_t^M \).

### 4.2.2 Excess of land supply

When there is excess of supply at a given time \( t^* \). At time \( t^* + 1 \) the dynamics of migration are different, given that the absentee owners will not be able to obtain the same amount from renting the land as before and workers produced more efficiently at time \( t^* \). Therefore there are two opposite effects on the modernization. The negative effect is due to the fact that owners that are not successful in the M-sector are obliged to migrate into the T-sector, the positive effect is that now all individuals in the T-sector can produce efficiently.

The following table summarizes this dynamic for each owner or worker:

<table>
<thead>
<tr>
<th>( t, t-1 )</th>
<th>T-sector [ Unsuccessful ]</th>
<th>M-sector [ Unsuccessful ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-sector</td>
<td>[ Unsuccessful ]</td>
<td>[ Unsuccessful ]</td>
</tr>
<tr>
<td>M-sector</td>
<td>[ Successful ]</td>
<td>[ Successful ]</td>
</tr>
</tbody>
</table>

The dynamic equations for the owners are:

\[
\omega_t^T = (1 - \alpha)O \quad \text{and} \quad o_t^M = \alpha O
\]  

(15)

while for the workers:

\[
u_t^T = (1 - \alpha)(1 - O) \quad \text{and} \quad u_t^M = \alpha (1 - O).
\]

(16)

### 4.3 Equilibria in the rental market

Given the dynamic equations for \( o_t^M, o_t^T, w_t^M \) and \( u_t^T \) in the two state (excess of demand and excess of supply at time \( t-1 \)) we are able to determine supply and demand at \( t \). As a result the dynamic version of the rental land demand of equation (4) is:

\[
D_t = 1 - M - n_t^N
\]

(17)

and the dynamic version of the static land rental supply (3) is:

\[
S_t = L - m_t^R
\]

(18)
Therefore, substituting the dynamic equations for $w_l^t$ and $c_l^t$ in (18) and (17) respectively we have the following expressions (in Appendix the analytical form):

$$D_t = \begin{cases} d_0(O, L) + d_1(O, L, \omega^o, \omega^w, t) & D_{t-1} > S_{t-1} \\ (1 - \alpha)(1 - O) & D_{t-1} < S_{t-1} \end{cases}$$

(19)

and

$$S_t = \begin{cases} s_0(O, L) + s_1(O, L, \omega^o, t) & D_{t-1} > S_{t-1} \\ L - (1 - \alpha)O & D_{t-1} < S_{t-1} \end{cases}$$

(20)

Given these expressions we are able to study the long run behavior of the economy and we have the following:

**Proposition 2** Given an initial wealth and land distribution $(L, O, \omega^o, \omega^w)$ i) $L > \frac{4 - \alpha^3 (2 - \alpha) - 2 \alpha (3 + O) + \alpha^2 (4 + O)}{4 - 4 \alpha + 3 \alpha^2 - \alpha^3}$ is a sufficient condition for the excess of land supply in the long run. ii) $L > (1 - \alpha)$ is a necessary and sufficient condition for the excess of land supply in the long run if for a given time $t_0$ it is true that $S_{t_0} > D_{t_0}$.

**Proof:**

Part i):

Let us consider first the case: $D_0 > S_0$. If $\lim_{t\to\infty} D_t (D_{t-1} > S_{t-1}) < \lim_{t\to\infty} S_t (D_{t-1} > S_{t-1})$ (see appendix for their analytical expressions) given that the two expression exists for all $t \in \{0, 1, \ldots\}$ it exists a $t^*$ such that $D_{t^*} (D_{t^*-1} > S_{t^*-1}) < S_{t^*} (D_{t^*-1} > S_{t^*-1})$. Passing at the limits of the two equations and comparing the two asymptotic values we have that it exist a $t^*$ such that $D_{t^*} < S_{t^*}$ if:

$$L > \frac{4 - \alpha^3 (2 - \alpha) - 2 \alpha (3 + O) + \alpha^2 (4 + O)}{4 - 4 \alpha + 3 \alpha^2 - \alpha^3}$$

(21)

Since $\frac{4 - \alpha^3 (2 - \alpha) - 2 \alpha (3 + O) + \alpha^2 (4 + O)}{4 - 4 \alpha + 3 \alpha^2 - \alpha^3} > 1 - \alpha$ for all $O \in [0, L]$ and $\alpha \in (0, 1)$, it is true that $L > 1 - \alpha$. At time $t^* + 1$ there will be $\alpha$ successful individuals in both the sector. This is true because the contract at $t^*$ is efficient and the probability of success in both sectors is $\alpha$. Therefore there will be $1 - \alpha$ poor individuals who want to work in the T-sector. Therefore if $L > 1 - \alpha$ there will be excess of supply also at time 1 and, consequently, given that the same reasoning applies for time 2, 3, ..., also in the long run. Therefore if $\lim_{t\to\infty} D_t (D_{t-1} > S_{t-1}) < \lim_{t\to\infty} S_t (D_{t-1} > S_{t-1})$ it has to be true that also $\lim_{t\to\infty} D_t < \lim_{t\to\infty} S_t$.

Now we prove that if (21) is not true $\lim_{t\to\infty} D_t > \lim_{t\to\infty} S_t$. Let us define the following function: $E_t = D_t (D_{t-1} > S_{t-1}) - S_t (D_{t-1} > S_{t-1})$. The following system has to be always verified for all $\alpha \in (0, 1)$ and for $t = 1, 2, \ldots, \infty$:
Substituting the analytical expression for $H$, $V$, and $G$ it is possible to show that (22) is true for all $\alpha$ and $\omega$. Case $S_0 > D_0$ is true for the part ii) of the proposition given that $\frac{4-\alpha^2 (2-\omega)+2 \alpha (3+\omega)+\alpha^2 (4+\omega)}{4-4 \alpha+3 \alpha^2-\omega^2} > 1 - \alpha$.

Part ii) :

Like we saw above if $L > 1 - \alpha$ and for a given $t_0$ it is $S_{t_0} > D_{t_0}$ then $\lim_{t \to \infty} S_t > \lim_{t \to \infty} D_t$. Now let us suppose that $L < 1 - \alpha$ if in a given time $t_0$ it is $S_{t_0} > D_{t_0}$ at time $t = t_0 + 1$, $S_t > D_t$ because $\alpha$ is the total number of rich individuals. Since $L < 1 - \alpha$ condition (21) is not satisfied therefore $\lim_{t \to \infty} S_t < \lim_{t \to \infty} D_t$.

The function $L(O) = \frac{4-\alpha^2 (2-\omega)+2 \alpha (3+\omega)+\alpha^2 (4+\omega)}{4-4 \alpha+3 \alpha^2-\omega^2}$ corresponds at the line $ab$ of figure 2. The area above this line represents the set of values of $O$ and $L$ such that there is an excess of supply of land in a given long run equilibrium for whatever initial wealth distribution. This line has a negative slope because for an higher $O$ the quantity of the land necessary to the economy to move to the second equilibrium is lower. The reason is simple: the greater the number of the owners, the greater the number of individuals migrating to the modern sector. This happens for two reason: i) they can more efficiently produce and have a higher probability of being successful; ii) when they are in the M-sector, they have a higher probability than the landless workers to stay in this sector because they can obtain the rent from their land. In other words, even if they are not successful in the M sector, there is a positive probability that their tenant is successful. This double task of the land can usefully related to the one considered in Kiyotaki and Moore (1997). In this paper the land is also a collateral for the credit market. In our paper we focus on the role of land as a form of insurance.

The area between $ab$ and $cd$ represents the area in which the equilibrium with excess of land supply is still possible but the initial wealth-land distribution has to be such that:

$$S_0 > D_0$$

or substituting the two terms and rearranging:

$$L > O(1 - \omega^p) + (1 - \omega^m)(1 - O)$$

in other words the initial number of poor in the economy has not to be too great.

When the demand exceeds the supply, there are two opposite effects: the positive level of rent increases the number of owners in the M-sector while the subefficiency of the contract decreases the mass of workers in the same sector. This happens if the mass of individuals that have to work in the T-sector will be greater than the total quantity of land in the economy. When the supply overcomes the demand, the amount of rent will fall to zero and the unsuccessful owners cannot finance their staying in the M-sector.
anymore and they go back to work on their own land. On the contrary workers can produce more efficiently in the T-sector, their probability of being successful increases and their demand for land will decrease. This situation of excess of supply is sustainable in the long run if the total quantity of land is large enough, namely \( L > 1 - \alpha \) (area above \( cd \)).

The area below \( cd \) represents the economy in which in the long run there will never be excess of land supply and the contracts will always be inefficient.

### 4.4 Modernization

From proposition 1 we know that when \( L < 1 - \alpha \) the economy cannot be in an equilibrium with excess of land supply in the long run. In this case the number of individuals migrating in the M sector, is:

\[
\omega^M_{\infty} + w^M_{\infty} = \frac{\alpha L}{2 - 2\alpha} + \frac{\alpha (2 + \alpha) O}{2 (2 - \alpha + \alpha^2)} \tag{25}
\]

This equation is represented in the first graph of figure 3. Like we said before the
larger the number of owners is the larger is the total number of individuals migrating to the modern sector. In this situation the land demand always exceeds the supply.

When the quantity of land per individuals is \(1 - \alpha \leq L < L^2\) condition (21) is not satisfied for all \(O\), therefore the economy is never in the equilibrium with “high modernization” unless the initial number of rich individuals is such that \(\omega^O \omega^w (1 - O) > L\). In this case we can see from the right graphic of figure 3 that the level will permanently be at its maximum.

An equilibrium with an excess of land supply has an higher efficiency of the T-sector and a larger number of individuals in the M-sector than the equilibrium with excess of land demand. When tenancy contracts become more efficient there there are two opposite effects on the migration. On one side the higher efficiency allow the workers to become rich with an higher probability, on the other side the absentee owners cannot finance their production with the rent obtained from their tenants. Like figure 3 shows us the positive effect overcomes the negative and the long run modernization will be higher.

Let us consider the area \(L^1 < L \leq L^2\). From figure 4 we can see that a more equalitarian land distribution has the effect of leading the economy to the more efficient equilibrium and to attain the maximum level of modernization. This is true for whatever level of initial wealth \(\omega^o\) and \(\omega^w\). Moreover also a wealth distribution with an high number of rich individuals has this same effect.

Finally let us consider a land rich economy with \(L > L^2\). In this case the land supply is always larger than its demand and the number of individuals in the M-sector is always at its maximum level.
5 Economic policy

In this section we will see how an agrarian reform which increases $O$ and how a one shot wealth redistribution that increases $\omega^o$ and $\omega^w$ affect the internal efficiency of the $T$-sector and the modernization of the economy.

When $L < 1 - \alpha$, we can observe in Figure 3 that an agrarian reform which increases $O$ have the effect of increasing the investor in the $M$-sector but not of leading the economy to the more efficient equilibrium. This long run equilibrium is totally independent from the initial portion of rich individual, therefore transferring a lump sum to poor individuals to affect the wealth distribution does not have consequences in the long run.

After the agrarian reform all successful dynasties will leave the $T$-sector and they sign an inefficient (because the demand exceeds the supply) sharecropping contract with workers. Therefore in the period after the agrarian reform its impact on the total efficiency of the agrarian sector becomes smaller. In figure 5 we can see the dynamic effect of an agrarian reform that at the end of a given time $t$ distribute $O_2 - O_1$ plot of land to a same measure of poor individuals. The line $0E$ represents the long run relationship between total owners and owners in the $T$-sector. This agrarian reform increases the number of owner workers of a measure $O_2 - O_1$ at time $t+1$. At time $t+2$ the number of owners in the same sector will start to decrease and in the long run the system will reach the equilibrium $E^{**}$. As we can see from figure 5 the number of workers in this sector will be $o_{2,1}^o < o_{2,2}^o \leq \ldots \leq o_{2,\infty}^o$. Therefore in the following periods an agrarian reform increases the number of owners in the $T$-sector of a lower number than the dynasties who receive the land.

In the right graphic of figure 3 we consider the economy when $1 - \alpha < L \leq L^2$. 
The agrarian reform has exactly the same effect than in the case before. However in this case a wealth redistribution can push the economy to the more efficient equilibrium with the maximum level of modernization (level $\alpha$ in figure 3). In this case the wealth redistribution has a stronger positive effect than the agrarian distribution in both sectors.

When $L^2 < L \leq L^1$ both agrarian reform and wealth distribution have the same effect of bringing the economy in the equilibrium with excess of supply and therefore to maximize its modernization. Wealth redistribution increases the size of the M sector and the efficiency of the tenancy contracts; agrarian reform which increases the number of owners $O \geq O^*$ brings the economy to the equilibrium with “high modernization”.

Finally (see figure 4) when the land is abundant tenurial contracts are always efficient. Consequently there is no way to improve the efficiency by fostering the process of migration, which is then at its maximum level. Therefore when the economy is rich of land a policy of redistribution (either of land or of monetary wealth) does not raise the efficiency of the economy.

6 Final remarks

Developing economies are normally characterized by a rural traditional sector with a
low efficiency in the production and a modern more efficient sector. In this paper we considered that the wealth constraints hampers the process of transition to the modern economy. Nevertheless we claim that the production in the traditional sector can help the economy to overcome this barrier. The main goal of this paper is to show that changes in traditional sectors have important impact also in the modern sector and, consequently, on the process of modernization of the economy.

Therefore economic policies have to take account of their impact on both the sectors. In particular we saw that an agrarian reform in some cases has the effect of fostering the modernization of the economy even if the effect on the traditional sector is less important than expected in the long run. On the contrary, in land rich economies, agrarian reform is totally ineffective because the price of renting the land and the consequent loss of efficiency is low.

Moreover our model allowed us to distinguish two kinds of wealth redistribution: of land and of monetary wealth. We showed that this two kinds of policies have different impact on the whole economy according to the ratio of land to individual. In an economy where this ratio is low agrarian reforms are better than monetary wealth distribution. In economies where this relation is not too low a monetary wealth distribution can be preferable. Finally in the economy rich of land with respect to people the two policies are ineffective.

References


**Appendix A. The tenancy contract**

We consider the contract when the land supply is larger than the demand and workers have all the bargaining power.

The problem for the workers is:

\[
\max_{c, v_w} e(1 - s) - c + v_w - \frac{c^2}{2\alpha} \quad \text{(P)}
\]

subject to:

\[
e(s)s = v_w \quad \text{(A-1)}
\]

\[
v_w \in \{0, u\} \quad \text{(A-2)}
\]

Maximizing the first equation for the effort:

\[
e^*(s) = \alpha(1 - s) \quad \text{(A-3)}
\]

Substituting \( e^* \) in (A-1) we have:
\[ \alpha(1 - s)^2 - \frac{\alpha(1 - s)^2}{2} + \nu_w \]  
(A-4)

From (A-1) we substitute \( \nu_w \) in (A-4). In that way we obtain:

\[ \alpha(1 - s)^2 - \frac{\alpha(1 - s)^2}{2} + \alpha(1 - s)s \]  
(A-5)

Maximizing for \( s \):

\[ -\alpha(1 - s) + \alpha(1 - 2s) = 0 \]  
(A-6)

or:

\[ s^* = 0 \]  
(A-7)

and from A-1:

\[ \nu_w = 0 \]  
(A-8)

Therefore workers prefer a first efficient contract without ex-post payment and do not want to become owners.

**Appendix B. Solution of the dynamic system**

We start to find the dynamic form of the equation \( \sigma_t^j \) and \( u_t^j \) that \( D_{t-1} > S_{t-1} \). Let us write equation (14) in the form:

\[ a \sigma_{t+2}^j - b \sigma_{t+1}^j + c \sigma_t^j = 0 \]  
(B-1)

where:

\[ a = \frac{2}{2 - 3 \alpha + \alpha^2} \]

\[ b = \frac{1 + \alpha}{1 - \alpha} \]

\[ c = \frac{\alpha}{2 - \alpha} \]

The solutions of the characteristic equation of (B-1) are:

\[ s = \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \]  
(B-2)
substituting a, b and c:

\[ s1 = \frac{(1 - \alpha) \alpha}{2} \]  \hspace{1cm} (B-3)

\[ s2 = \frac{1}{2} \]  \hspace{1cm} (B-4)

Given an initial wealth-land distribution \( \{ \hat{L}, \hat{O}, \omega^o, \omega^w \} \) The initial conditions are:

\[ n_0^T = (1 - \omega^o) \hat{O} \] and \[ n_1^T = (1 - \alpha) \hat{O} (1 - \omega^o) + (1 - \alpha) \hat{O} \omega^o (1 - \frac{\alpha}{2}) \], we have the following system solved by the two coefficients of the solution of (B-1):

\[ c_1 + c_2 = n_0^T \]

\[ c_1 \frac{(1 - \alpha) \alpha}{2} + c_2 = n_1^T. \]

Consequently we have:

\[ c_1 = \hat{O} \left( \frac{2 - 3 \alpha + \alpha^2}{2 - \alpha - \alpha^2} + \omega^o \right) \]

and

\[ c_2 = \frac{(2 - 3 \alpha + \alpha^2) \hat{O}}{2 - \alpha + \alpha^2} \]

Therefore the solution of (B-1) is:

\[ \phi^T_i = \frac{(2 - 3 \alpha + \alpha^2) \hat{O}}{2 - \alpha + \alpha^2} + \frac{(1 - \alpha) \alpha^i \hat{O} \left( \frac{2 - 3 \alpha + \alpha^2}{2 - 2 + \alpha - \alpha^2} + \omega^o \right)}{2^i} \]  \hspace{1cm} (B-5)

Considering that:

\[ \phi^M_i = \phi^T_i - \phi^T_{i+1} \]  \hspace{1cm} (B-6)

We have:

\[ \phi^M_i = \frac{2 \alpha \hat{O}}{2 - \alpha + \alpha^2} - \frac{(1 - \alpha) \alpha^i \hat{O} \left( \frac{2 - 3 \alpha + \alpha^2}{2 - 2 + \alpha - \alpha^2} + \omega^o \right)}{2^i} \]  \hspace{1cm} (B-7)

If we substitute the equation for \( \phi^M_i \) in equation (13) we have:

\[ \phi^T_i = \hat{L} - \frac{(2 - 3 \alpha + \alpha^2) \hat{O}}{2 - \alpha + \alpha^2} - 2^{1-i} (1 - \alpha)^i \alpha^i \hat{O} \left( \frac{2 - 3 \alpha + \alpha^2}{2 - 2 + \alpha - \alpha^2} + \omega^o \right) \]  \hspace{1cm} (B-8)

finally we substitute this last expression in (12) and we obtain a difference equation in the form:

\[ w_i^M = a w_{i-1}^M + b + cd^i \]  \hspace{1cm} (B-9)

with:

22
\[ a = \alpha \]  
\[ b = \frac{\alpha \left( L - \frac{(2 - 3\alpha + \alpha^2) O}{2 - \alpha + \alpha^2} \right)}{2} \]  
\[ c = \frac{-\left( \alpha O \left( \frac{2 - 3\alpha + \alpha^2}{2 - \alpha + \alpha^2} + \omega^o \right) \right)}{2} \]

and

\[ d = \frac{(1 - \alpha) \alpha}{2}. \]  

Solving recursively (B-9) and substituting the expressions for \( a, b, c \) and \( d \), we have:

\[ w_i^M = \alpha \left( L - \frac{(2 - 3\alpha + \alpha^2) O}{2 - \alpha + \alpha^2} \right) + \]  
\[ \alpha^t (1 + \frac{\alpha L}{2 - 2\alpha}) - n + O \left( -1 - \frac{(-2 + \alpha) \alpha}{2 (2 - \alpha + \alpha^2) + \omega^o} \right) \]  
\[ \alpha^t \left( 1 - \frac{(1 - \alpha)^t O}{2} \right) \frac{2 - 3\alpha + \alpha^2}{2 + \alpha + \alpha^2} - \frac{\omega^o}{1 + \alpha} \]  

If we substitute \( w_i^M \) and \( \alpha^t \) in (18) and (17) we have:

\[ S_i^{D_i \to S_{i-1}} = L - \frac{(2 - 3\alpha + \alpha^2) O}{2 - \alpha + \alpha^2} - \frac{(1 - \alpha)^t O \left( \frac{2 - 3\alpha + \alpha^2}{2 - \alpha + \alpha^2} + \omega^o \right)}{2} \]  

and

\[ D_i^{D_i \to S_{i-1}} \]

\[ = 1 - O - \frac{\alpha \left( L - \frac{(2 - 3\alpha + \alpha^2) O}{2 - \alpha + \alpha^2} \right)}{2 (1 - \alpha)} + \]  
\[ \frac{(1 - \alpha)^t O \left( \frac{2 - 3\alpha + \alpha^2}{2 - \alpha + \alpha^2} - \frac{\omega^o}{1 + \alpha} \right)}{2} + \]  
\[ \alpha^t \left( \frac{\alpha^2 (2 + L - 3O - 2\omega^w + 2O\omega^w)}{2 (-1 + \alpha^2)} + \alpha (L + O (3 - \omega^o)) + 2 (-1 + \omega^w + O (-\omega^w + \omega^o)) \right) \]  

\[ \frac{2 (-1 + \alpha^2)}{2 (-1 + \alpha^2)} \]
Which are the analytical expression of the first equations of expressions (20) and (19).