Fourth Summer School in Trade, Industrialisation, and Development 2005

Gargnano, Italy

Trade, Innovation, and Technology Diffusion: Implications for Developing Countries

Lecture 5: Innovation and Diffusion

September 2005

The Value of an Idea

1. With profit Π and T ideas, the expected profit to any particular idea is just:

$$h(t) = \frac{\Pi(t)}{T(t)} = \frac{\delta^i X(t)}{T(t)},$$

with $\delta^i \in \left\{\delta^{BC}, \delta^{MC}\right\}$ depending on the form of competition, or:

$$h(t) = \frac{\delta^i}{1-\delta^i} \frac{wL^P(t)}{T(t)}.$$

2. Numeraire w. Exogenous discount rate $\rho > 0$.

3. Discounted value of an idea:

$$V(t) = \int_t^\infty e^{-\rho(s-t)} h(s) \frac{P(t)}{P(s)} ds.$$

4. **Bertrand Competition**

$$V^{BC}(t) = \frac{\delta^{BC}}{1-\delta^{BC}} w T(t)^{-1/\theta} \int_t^\infty e^{-\rho(s-t)} L^P(s) T(s)^{(1-\theta)/\theta} ds$$

5. **Monopolistic Competition**

$$V^{MC}(t) = \frac{\delta^{MC}}{1-\delta^{MC}} w T(t)^{-1/\theta} \int_t^\infty e^{-\rho(s-t)} \left[L^P(s)\right]^{(\theta\overline{m}-1)/\theta]} T(s)^{(1-\theta)/\theta} ds$$

The Creation of Ideas

1. Idea Production Function:

$$dT(t) = R(t)dt = \alpha(t)r(t)^{\beta}L(t)dt$$

   (a) $\alpha(t)$:research productivity

   (b) $L(t)$ = measure of workers

   (c) $r(t)$ = share engaged in research

   (d) $\beta \in [0, 1]$ parameter of diminishing returns to research.

2. Labor-Market Equilibrium: :

$$
\begin{aligned}
\beta\alpha(t)V(t)r(t)^{\beta-1} &= w \quad r(t) \in [0,1] \\
\beta\alpha(t)V(t)r(t)^{\beta-1} &< w \quad r(t) = 0 \\
\beta\alpha(t)V(t)r(t)^{\beta-1} &> w \quad r(t) = 1
\end{aligned}
$$

3. $L(t)$ grows at rate $n \geq 0$.

4. Inventor owns idea (Perfect patent protection)

Steady-State Growth

1. g_T, r constant

$$g_T = \alpha(t) r^{\beta} \frac{L(t)}{T(t)}$$

Endogenous Growth

1. $n = 0$ so $L(t) = L$, $\alpha(t) = \alpha T(t)$.

2.

$$g_T = \alpha r^{\beta} L.$$

3.

$$V(t) = \Gamma^{i} \frac{w(1-r)L}{T(t)} \frac{1}{\rho\theta + (\theta - 1)\alpha r^{\beta} L}$$

(a) Bertrand competition $\Gamma^{BC} = 1$

(b) Monopolistic Competition

$$\Gamma^{MC} = \frac{\theta}{\theta \overline{m} - 1}$$

4. Result: Interior labor-market equilibrium condition:

$$\beta\Gamma^i - \theta\Lambda r^{1-\beta} = (\beta\Gamma^i + \theta - 1)r$$

where:

$$\Lambda = \frac{\rho}{\alpha L}.$$

5. Implication: r, and hence g_T, fall with Λ.

6. Special case: Constant Returns to Scale in research ($\beta = 1$).

(a) **Bertrand competition:**

$$\begin{aligned} r &= \frac{1}{\theta} - \frac{\rho}{\alpha L} \\ g_T &= \frac{\alpha L}{\theta} - \rho \end{aligned}$$

$r \in [0, 1]$.

(b) **Monopolistic competition:**

$$
\begin{aligned}
r &= \frac{\theta}{1+\theta\overline{m}(\theta-1)} - \frac{\theta\overline{m}-1}{1+\theta\overline{m}(\theta-1)}\frac{\theta\rho}{\alpha L} \\
g_T &= \frac{\theta}{1+\theta\overline{m}(\theta-1)}\alpha L - \frac{\theta\overline{m}-1}{1+\theta\overline{m}(\theta-1)}\theta\rho
\end{aligned}
$$

$r \in [0,1]$.

Semi-Endogenous Growth

1. $n > 0$, $n < \rho$, $\alpha(t) = \alpha$.

2. $$g_T = \alpha r^{\beta} / \tau(t),$$

   where $\tau(t) = T(t)/L(t)$

3. Steady state with τ constant $\longrightarrow g_T = n$.

4. Since

$$\begin{aligned} \dot{\tau} &= \alpha r^{\beta} - \tau n, \\ \tau &= \frac{\alpha r^{\beta}}{n} \end{aligned}$$

5. Prices decline faster with monopolistic competition since $n > 0$.

6. **Bertrand Competition:**

$$V^{BC} = \frac{w(1-r)}{\alpha r^{\beta}} \frac{1}{(\rho\theta/n) - 1}.$$

$$\begin{aligned} r &= \max\left[0, \frac{\beta}{(\rho\theta/n) - (1-\beta)}\right] \\ &= \frac{n}{\rho\theta} \qquad (\beta = 1) \end{aligned}$$

7. **Monopolistic Competition:**:

$$V^{MC} = \frac{\delta^{MC}}{1-\delta^{MC}} \frac{w(1-r)}{\alpha r^{\beta}} \frac{1}{(\rho/n) - (\overline{m}/\sigma)}$$

$$r = \max\left[0, \frac{\beta/(\theta\overline{m}-1)}{(\rho/n) - (\overline{m}/\sigma) + \beta/(\theta\overline{m}-1)}\right].$$

$r \in [0, 1]$

8. Result: Research effort and the level of income increase with n/ρ.

Diffusion

1. Diffusion lag is a random variable τ_{ni} that is exponentially distributed with parameter $\epsilon_{ni} \geq 0$; that is:

   $$\Pr[\tau_{ni} \leq t] = 1 - \exp(-\epsilon_{ni} t).$$

   with mean $1/\epsilon_{ni}$

2. Implication

   $$\begin{aligned} dT_n(t) &= \sum_{i=1}^{N} \epsilon_{ni} \int_{-\infty}^{t} \exp[-\epsilon_{ni}(t-s)] R_i(s) ds \\ &= \sum_{i=1}^{N} \epsilon_{ni} \int_{-\infty}^{t} \exp[-\epsilon_{ni}(t-s)] \alpha_i(s) r_i(s)^{\beta} L_i(s) ds \end{aligned}$$

3. No international trade in the goods produced using ideas generated by research, but allow inventors to earn the return to their inventions that accrues abroad. We assume that these goods can be aggregated into a final good that is produced competitively and can be traded costlessly.

4. Value of idea from i in n:

$$V_{ni}(t) = \int_t^{\infty} \mathsf{exp}[-\rho(s-t)] \left\{1 - \mathsf{exp}[-\epsilon_{ni}(s-t)]\right\} \frac{\delta^k X_n(s)}{T_n(s)} \frac{P(t)}{P(s)} ds$$

where $k = BC, MC$.

5. Value of idea from i:

$$V_i(t) = \sum_{n=1}^{N} V_{ni}(t)$$

Here: Bertrand Case with Endogenous Growth

1. Country i: Fixed labor force L_i, research productivity $\alpha_i(t) = \alpha_i T_i(t)$, $\alpha_i \geq 0$

2. Steady state: constant r_i and common g_T.

3. System of linear differential equations:

$$\dot{T}(t) = \Delta T(t)$$

where Δ has representative element:

$$\Delta_{ni} = \frac{\epsilon_{ni}}{\epsilon_{ni+} g_T} \alpha_i r_i^{\beta} L_i,$$

4. Indecomposability: We can't write order countries to write $\mathbf{\Delta}$ as:

$$\mathbf{\Delta} = \begin{bmatrix} \mathbf{\Delta}_{AA} & \mathbf{\Delta}_{AB} \\ \mathbf{0} & \mathbf{\Delta}_{BB} \end{bmatrix}.$$

where $\mathbf{\Delta}_{AA}$ and $\mathbf{\Delta}_{BB}$ are square matrices.

5. Result (Frobenius): If $\mathbf{\Delta}$ is indecomposable then there is a common positive growth rate g_T that is increasing in each element of $\mathbf{\Delta}$. Associated with that g_T is a vector T defined up to a scalar multiple that reflects each countries relative stock of ideas.

6. $N = 2$, normalizing $T_1 = 1$:

$$\begin{aligned} g_T &= \frac{\Delta_{11} + \Delta_{22}}{2} + \frac{\sqrt{(\Delta_{22} - \Delta_{11})^2 + 4\Delta_{12}\Delta_{21}}}{2} \\ T_2 &= \frac{\Delta_{22} - \Delta_{11}}{2\Delta_{12}} + \frac{\sqrt{(\Delta_{22} - \Delta_{11})^2 + 4\Delta_{12}\Delta_{21}}}{2\Delta_{12}}. \end{aligned}$$

strictly increasing in each country's research and in all four rates of diffusion.

7. Solving for r:

$$\frac{w_i}{w_{i'}} = \left(\frac{T_i}{T_{i'}}\right)^{1/\theta}$$

8.

$$V_i = \frac{\delta^k}{1-\delta^k} \sum_{n=1}^{N} K_{ni} \frac{(1-r_n)L_n}{T_n(t)}$$

where:

$$K_{ni} = \frac{1}{\rho + (1-1/\theta)g_T} - \frac{1}{\rho + (1-1/\theta)g_T + \epsilon_{ni}}$$

9. Solution for r:

$$r_i^{1-\beta} = \alpha_i \beta \frac{\delta^k}{1-\delta^k} \sum_{n=1}^{N} K_{ni}(1-r_n) L_n \left(\frac{T_n(t)}{T_i(t)} \right)^{(1-\theta)/\theta}$$

requiring numerical solution.