Fourth Summer School in Trade, Industrialisation, and Development 2005

Gargnano, Italy

Trade, Innovation, and Technology Diffusion: Implications for Developing Countries

Lecture 2: Modeling the Fundamentals of Technology, Innovation, and Trade

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Introduction

A basic structure of technology and cost heterogeneity underlying a great deal of current work. (EK, BEJK, EKK; Melitz, HMY, HMR, Chaney)

Goal: A framework that can integrate producer level heterogeneity with aggregate measurements.

Ideas and Locations

1. Idea: How to produce some good j with efficiency q; $j \in [0, J]$.

2. Efficiency q: the realization of a random variable drawn from the Pareto distribution with parameter θ and lower bound $\underline{q}$ (q: "quality" of idea).

3. Measure of ideas with quality $Q \geq q \geq \underline{q}$.

$$\Pr[Q > q] = \left(\frac{q}{\underline{q}}\right)^{-\theta}$$

4. Arrival rate at location i of ideas for good j at time t better than $Q \geq q$:

$$\overline{a} R_i(j,t) \left(\frac{q}{\underline{q}} \right)^{-\theta}$$

5. Normalize $\overline{a}\underline{q}^{-\theta} = 1$ and let $\underline{q} \longrightarrow 0$.

6. Location i:'s history of arrival of ideas good j better than quality q: $T_i(j,t)q^{-\theta}$ where:

$$T_i(j,t) = \int_{-\infty}^{t} R_i(j,\tau) d\tau$$

7. Number of ideas at location i about good j with quality better than q is distributed Poisson with parameter $T_i(j,t)q^{-\theta}$

8. Location i: input cost w_i.

9. Location i determined by $T_i(j,t)$, w_i and, later, geography. Forget i for now.

Techniques and unit costs

1. Unit cost $C = w_i/Q$

$$F(c) = \mathsf{Pr}[C \le c] = \mathsf{Pr}\left[Q \ge \frac{w_i}{c}\right] = \left(\frac{w_i}{c\underline{q}}\right)^{-\theta}$$

2. Number of techniques with unit cost $C \le c$ is Poisson with parameter:

$$\Phi_i(j,t)c^\theta$$

3. Techniques for good j ordered by unit cost $C_i^{(1)}(j,t) \le C_i^{(2)}(j,t) \le C_i^{(3)}(j,t) \le \ldots.$

Distributional Results

Results on the joint distribution of $C^{(k)}$, which depend only on the two parameters $\Phi_i(j,t)$ and θ:

1. Given n ideas, the probability $P(c_k, c_{k+1}|n)$ that $C^{(k)} \leq c_k \leq c_{k+1} \leq C^{(k+1)}$:

$$\frac{n!\,[F(c_k)]^k\,[1-F(c_{k+1})]^{n-k}}{(k-1)!(n-k-1)!}$$

an object closely related to the joint distribution of $C^{(k)}$ and $C^{(k+1)}$.

2. Taking the negative of the cross derivative with respect to c_k and c_{k+1} gives the joint density::

$$g_{k,k+1}(c_k, c_{k+1}|n) = \frac{n!\,[F(c_k)]^{k-1}\,[1 - F(c_{k+1})]^{n-k-1}\,dF(c_k)dF(c_{k+1})}{(k-1)!(n-k-1)!},$$

(Equation 3, Section 4.6, Hogg and Craig, 1995).

3. Since the number n is a Poisson draw with parameter $\overline{a}T$ the expectation of this joint distribution unconditional on n is

:

$$
\begin{aligned}
g_{k,k+1}(c_k, c_{k+1}) &= \sum_{n=0}^{\infty} \frac{\exp(-\overline{a}T)\,(\overline{a}T)^n}{n!} g_{k,k+1}(c_k, c_{k+1}|n) \\
&= \frac{[F(c_k)]^{k-1}\,(\overline{a}T)^{k+1}\exp[-\overline{a}TF(c_{k+1})]dF(c_k)dF(c_{k+1})}{(k-1)!} \\
&\qquad \sum_{n=k+1}^{\infty} \frac{e^{-\overline{a}T[1-F(c_{k+1})]}\left\{\overline{a}T\left[1-F(c_{k+1})\right]\right\}^{n-k-1}}{(n-k-1)!} \\
&= \frac{[F(c_k)]^{k-1}\,(\overline{a}T)^{k+1}\exp[-\overline{a}TF(c_{k+1})]dF(c_k)dF(c_{k+1})}{(k-1)!} \\
&\qquad \sum_{m=0}^{\infty} \frac{e^{-\overline{a}T[1-F(c_{k+1})]}\left\{\overline{a}T\left[1-F(c_{k+1})\right]\right\}^{m}}{m!} \\
&= \frac{[F(c_k)]^{k-1}\,(\overline{a}T)^{k+1}\exp[-\overline{a}TF(c_{k+1})]dF(c_k)dF(c_{k+1})}{(k-1)!}
\end{aligned}
$$

The last result follows since the summation is over the domain of the Poisson distribution with parameter $\overline{a}T\left[1 - F(c_{k+1})\right]$. Substituting the Pareto distribution for $F(c)$ we get

$$g_{k,k+1}(c_k, c_{k+1}) = \frac{\theta^2}{(k-1)!}\Phi^{k+1} c_k^{\theta k-1} c_{k+1}^{\theta-1} \exp[-\Phi c_{k+1}^{\theta}]$$

for $0 < c_k \leq c_{k+1} < \infty$ while the marginal density of $C^{(k)}$ is:

$$g_k(c_k) = \frac{\theta}{(k-1)!}\Phi^{k} c_k^{\theta k-1} \exp[-\Phi c_k^{\theta}].$$

Results:

1. The distribution of the lowest cost $C^{(1)}$ for producing a good is:

$$F_1(c_1) = \Pr[C^{(1)} \leq c_1] = 1 - \exp\left[-\Phi c_1^{\theta}\right]$$

2. The moments of $C^{(1)}$ are given by (for $\theta + b > 0$):

$$E\left[\left(C^{(1)}\right)^b\right]^{1/b} = \left[\Gamma\left(\frac{\theta + b}{\theta}\right)\right]^{1/b} \Phi^{-1/\theta}.$$

3. The moments of $C^{(2)}$ are given by (for $2\theta + b > 0$):

$$E\left[\left(C^{(2)}\right)^b\right]^{1/b} = \Gamma\left(\frac{2\theta + b}{\theta}\right)^{1/b} \Phi^{-1/\theta}.$$

4. The ratio $M = C^{(2)}/C^{(1)}$ is independent of $C^{(2)}$ and is distributed:

$$F_{2/1}(m) = \Pr\left[M \le m\right] = 1 - m^{-\theta}.$$

5. Conditional on $C^{(1)} = c_1$, the distribution of $C^{(2)}$ is:

$$\Pr[C^{(2)} \le c_2 | C^{(1)} = c_1] = 1 - \exp\left[-\Phi(c_2^\theta - c_1^\theta)\right]$$

6. The distribution of the ratio $M = C^{(2)}/C^{(1)}$ given $C^{(1)} = c_1$ is:

$$\Pr\left[M \le m | C^{(1)} = c_1\right] = 1 - \exp\left[-\Phi c_1^\theta (m^\theta - 1)\right].$$

Preferences and Demand

Preferences: nested CES

1. lower tier: elasticity of substitution σ' across versions of a good $j \in [0, J]$ indexed by unit cost $c^{(k)}(j)$, $k = 1, 2, 3, ...$

2. upper tier: elasticity of substitution σ across goods $j \in [0, J]$.

3. $\sigma' \geq \sigma > 1$

4. Analysis above applies to each good $j \in [0, J]$

5. Convention: The realized distribution across the continuum of goods $[0, J]$ replicates the probabilistic distribution for each good. j

6. Here $J = 1$.

7. Spending on version k of good j with price $p^k(j)$ given total spending X:

$$X^k(j) = X\left(\frac{p^k(j)}{p(j)}\right)^{1-\sigma'}\left(\frac{p(j)}{P}\right)^{1-\sigma}.$$

where:

$$p(j) = \left[\sum_{k=1}^{L(j)} p^k(j)^{1-\sigma'}\right]^{1/(1-\sigma')}$$

the price index for good j, $L(j)$ the number of varieties of it offered.

8. Aggregate price index:

$$P = \left[\int_0^J p(j)^{1-\sigma} dj\right]^{1/(1-\sigma)}.$$

Market Structure and Ownership of Ideas

1. $\sigma' \to \infty$, perfect competition, common access to ideas delivers the Ricardian competitive model with a continuum of goods as in Eaton and Kortum (2002).

2. $\sigma' \to \infty$, Bertrand competition among proprietary owners of each idea delivers quality ladders as in Kortum (1997), Eaton and Kortum (1999), and BEJK (2003).

3. Setting $\sigma' = \sigma$ with proprietary owners of ideas and a fixed cost of production delivers monopolistic competition as in Melitz (2003), Helpman, Melitz, and Yeaple (2004), Chaney (2005), and Helpman, Melitz, and Rubinstein (2005) consider variants of this case.

4. General σ' and proprietary ownership of ideas with Cournot competition (Atkeson and Burstein, 2005).

5. Here $J = 1$.

I. Perfect Competition

1. $\sigma' \rightarrow \infty$

2. Common access to ideas

3. $p(j) = C^{(1)}(j)$

4. Price index

$$
\begin{aligned}
P &= \left[\int_0^1 C^{(1)}(j)^{1-\sigma} dj\right]^{1/(1-\sigma)} \\
&= \left[\int_0^\infty c_1^{1-\sigma} dF_1(c_1)\right]^{1/(1-\sigma)} \\
&= E\left[\left(C^{(1)}\right)^{1-\sigma}\right]^{1/(1-\sigma)} \\
&= \gamma^{PC} \Phi^{-1/\theta}.
\end{aligned}
$$

where

$$
\gamma^{PC} = \left[\Gamma\left(\frac{\theta - (\sigma - 1)}{\theta}\right)\right]^{1/(1-\sigma)}
$$

and $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$ is the Gamma function.

Bertrand Competition

1. $\sigma' \to \infty$

2. Price:

$$P(j) = \mathsf{min}\left\{\overline{m}C^{(1)}(j), C^{(2)}(j)\right\}$$

where

$$\overline{m} = \frac{\sigma}{\sigma - 1}$$

3. Markup:

$$M(j) = \frac{P(j)}{C^{(1)}(j)} = \mathsf{min}\left\{\frac{C^{(2)}(j)}{C^{(1)}(j)}, \overline{m}\right\}$$

4. Markup Distribution:

$$\Pr\left[M \le m\right] = F_M(m) = 1 - m^{-\theta}$$

for $m \le \overline{m}$.independent of $C^{(2)}$ otherwise $m = \overline{m}$.

5. Price index:

$$P = \gamma^{BC} \Phi^{-1/\theta}$$

where:

$$\gamma^{BC} = \left[1 + \frac{(\sigma - 1)\overline{m}^{-\theta}}{\theta - (\sigma - 1)} \Gamma\left(\frac{2\theta - (\sigma - 1)}{\theta} \right) \right]^{1/(1-\sigma)}.$$

6. Profit

$$\pi(j) = (P(j) - C^{(1)}(j))(X(j)/P(j)) = (1 - M(j)^{-1})X(j).$$

$$\Pi = \delta^{BC} X$$

where

$$\delta^{BC} = \frac{1}{1+\theta}$$

X = total spending. Note: even though the markup is capped at $\overline{m} = \sigma/(\sigma - 1)$, profit share is independent of σ.

A lower unit cost $C^{(1)}(j)$ is associated with:

1. A lower price, whether $\overline{m}C^{(1)}(j)$ or $C^{(2)}(j)$
2. With $\sigma > \mathbf{1}$, larger sales
3. A higher markup.

Monopolistic Competition

1. $\sigma' = \sigma$

2. Price $p^k(j) = \overline{m}C^{(k)}(j)$

3. Fixed cost $F > 0$ to serve the market

4. Variable profit of a firm with cost c and charging price p:

$$\Pi^V(c) = (p - c)X(j)/p.$$

5. Standard result:

$$p = \overline{m}c$$

6. Variable profit:

$$\Pi^V(c) = \frac{X(j)}{\sigma} = \frac{X}{\sigma}\left(\frac{\overline{m}c}{P}\right)^{1-\sigma}, \tag{1}$$

decreases in cost c.

7. Entry cutoff and price index:

$$\bar{c} = \left(\frac{X}{\sigma F}\right)^{1/(\sigma-1)} \frac{P}{\overline{m}}. \tag{2}$$

8. Price index and entry cutoff:

$$\begin{aligned} P &= \left[\int_0^{\bar{c}} (\overline{m}c)^{1-\sigma}\, dJ(c)\right]^{1/(1-\sigma)} \\ &= \overline{m}\left[\Phi \int_0^{\bar{c}} \theta c^{\theta-\sigma} dc\right]^{1/(1-\sigma)} \\ &= \overline{m}\left[\frac{\theta\Phi}{\theta-(\sigma-1)}\bar{c}^{\theta-(\sigma-1)}\right]^{1/(1-\sigma)} . \end{aligned} \tag{3}$$

9. Solution:

$$\overline{c} = \left(\frac{\theta - (\sigma - 1)}{\theta \Phi} \frac{X}{\sigma F} \right)^{1/\theta}$$

Price index:

$$P = \gamma^{MC} \left(\frac{X}{\sigma F} \right)^{-[\theta - (\sigma - 1)]/[(\sigma - 1)\theta]} \Phi^{-1/\theta}$$

where

$$\gamma^{MC} = \overline{m} \left[\frac{\theta}{\theta - (\sigma - 1)} \right]^{-1/\theta}$$

10. Measure of active sellers S:

$$S = \frac{X}{\sigma F}\frac{\theta - (\sigma - 1)}{\theta}.$$

Note:

1. The measure of sellers increases with respect to market size relative to the fixed cost (X/F) with an elasticity of one.

2. If F increases with X with elasticity ε (i.e., $F = fX^{\varepsilon}$, where f is some positive constant) the elasticity of entry with respect to market size X will then be $1 - \varepsilon$.

3. Hence the price level falls with respect to market size relative to the fixed cost (X/F) with an elasticity:

$$\frac{\theta - (\sigma - 1)}{\theta(\sigma - 1)}$$

4. But S is independent of $\Phi = Tw^{-\theta}$.

5. Hence, the price level relates to the level of technology as in the case of perfect competition and the quality ladders model, falling with respect to the measure of ideas T with elasticity $1/\theta$.

6. Average price P^S of goods actually sold:

$$\begin{aligned} P^S &= \left[\frac{1}{S}\int_0^{\overline{c}} (\overline{m}c)^{1-\sigma}\, dJ(c)\right]^{1/(1-\sigma)} \\ &= \overline{m}\left[\frac{\theta}{\theta-(\sigma-1)}\right]^{1/(1-\sigma)} \overline{c}. \end{aligned}$$

which increases in X/F. (Ghironi and Melitz).

7. Aggregate variable profit:

$$\Pi^V = \frac{X}{\sigma}$$

8. Aggregate profit:

$$\Pi = \delta^{MC} X$$

where

$$\delta^{MC} = \frac{\sigma - 1}{\theta \sigma}$$

9. average profit per producer Π/S is:

$$\frac{\sigma - 1}{\theta - (\sigma - 1)} F$$

Differences with Melitz and others: Profits survive. Important for growth later.